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THE DESIGN OF SPECIAL PURPOSE HORIZONTAL GEODETIC CONTROL NETWO--ETC(U)
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DEFENSE MAPPING AGENCY

TECHNICAL REPORT No. DMA 76-003

THE DESIGN OF SPECIAL-PURPOSE HORIZONTAL GEODETIC CONTROL NETWORKS

William H. Sprinsky
Lieutenant Colonel, USA

OCTOBER 1976

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DEFENSE MAPPING AGENCY
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The problem of the design of horizontal geodetic control networks and the choice of observations which will establish the network is examined from the standpoint of the final user specified positional accuracy and the required accuracies of user specified estimable quantities. A criterion variance covariance matrix is constructed solely from user requirements and predicted correlations from the propagated variance covariance matrix of possible establishing observations is used to choose those least correlated. An alternate approach is examined using the criterion

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variance covariance matrix and selected observations in a back solution for the observational accuracy required to recreate the positional parameter variance covariance matrix.

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FOREWORD

This dissertation was prepared while the author was pursuing doctoral studies at The Ohio State University, Columbus, Ohio. Since this subject matter is of interest to the DoD community, it is published as a Defense Mapping Technical Report. The author is presently serving as Chief, Programs, Productions, and Operations, Defense Mapping School (DMS); at the time of preparation he was Chief, Survey Department, DMS.

The general format is that prescribed for Ph.D. dissertations at The Ohio State University. In order to make this publication available to requestors without added costs, no format changes from those required to meet graduate publication specifications to DoD publication standards have been made.

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Without the computing facility of the Instruction and Research Computer Center of The Ohio State University and the advice of it's employees, none of the numerical evaluations of the concepts presented in this study would have been possible. The author is grateful for the access provided to this facility and it's personnel.

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Chapter 1. Introduction

1.1 Basic defnitions.

This study primarily concerns itself with the design of horizontal geodetic control networks. These networks are systems of points on the surface of the earth whose latitude and longitude are determined with respect to a datum by horizontal geodetic surveys, which provide observations that are adjusted to the applicable mathematical figure of the earth (U.S. Department of Commerce (1974)). In a sense, this study is a special application of the design of the experiments problem discussed by Federov (1972) and Hicks (1964) among others. The methods and principles developed will in every case be referred to horizontal control specifically, but the theory is applicable to many other design problems. "Design" in the sense used here is defined to mean the choice of geodetic observations which establish the horizontal network.

1.2 Design with respect to control point use.

Classically, observations were chosen for control surveys by reconnaissance parties, guided by intervisibility between proposed stations and a set of rules or specifications which determined the type of figure used and its configuration, for different orders of surveys. These orders of surveys and the numbers which determine the configuration of the observations are given in U.S.D.C. (1974). Nowhere in this referenced publication, or in others on the subject of horizontal

control surveys, is there mention of how accurately the position of stations will be determined if these specifications are met, except in the most general of terms, nor is there comment on the accuracy of such derived quantities as the distance between any two points in the net or the angle observed at a station, between the normal section planes containing any other two stations, which may be computed from the determined positions of the stations.

Yet quantities computed from station positions are those most used by surveyors who must base determinations of such things as plat boundaries, areas and route locations on basic survey control. Those who utilize the control data from the survey to be designed will be called as "users" and their requirements are based not only on positional accuracy but also on the accuracy of these computed quantities. For example, the artillery surveyor is much less concerned with the latitude and longitude of an artillery concentration point then he is with the distance and azimuth of that point with respect to the location of his batteries. The land surveyor, who is traversing to determine the route of a road by starting and ending his traverse on control points within the net to be designed is primarily interested in the distance and azimuth of a line joining the start and end points. This line (and not the geodetic positions of the start and end points) will be the basis for determination of the acceptability of his survey.

Since these points in the horizontal control net are marked by monuments which may be disturbed by either man or nature, it is common practice to try to verify that the monument has not, in fact, been moved before starting a supplementary survey in which the control point

is to be included. The position of this control point is not usually measured by the surveyor. He depends either on witness marks or check measurements whose values he can compute from geodetic network station location values. If, for example, his project specifications call for the measurement of the azimuth between two stations in the geodetic network to "check" their recovery and reoccupation, he compares what he measures with the value he computes. If the difference between what he measures and computes is large, he may conclude that the geodetic stations are unusable, when, in fact, the value he computes may have a very large uncertainty and the stations he is concerned with are perfectly recovered.

Special networks in the title of this study imply those where not only station location uncertainty but also the uncertainty of specific user supplied derived quantities are considered as part of the mission statement given the designer. It might be argued that, if this desire to reduce uncertainty is a part of the designer's mission, stations in the control net should be established with the least uncertainty possible considering the state of the surveyor's art at the time of the geodetic survey. One fact militates against this approach in design and this is cost of the geodetic survey. This cost can be reckonned on the basis of money required to perform the control survey, on man hours to be devoted to the job or on both. There is, therefore, a "trade-off" required between accuracy and cost. The designer, based on his knowledge of costs and experience, performs this "trade-off" in a non-mathematical manner, when he chooses observations which minimize costs and fit into

an efficient, time-phased plan. The algorithms for mathematically performing this task are complex at best, subjective and may not be general enough, if developed, to apply to more than one specific problem, crew, location and set of requirements.

The purpose of this study is to provide a method that assists the designer in this trade-off by mathematically determining which groups of observations, from all the observations possible to perform, are needed to meet user requirements and allow him, on the basis of cost, to choose which specific observation or observations from each group he will use.

1.3 Limitations in the Design method based upon approximate coordinates.

One of the requirements which the designer receives from the user is the number and approximate location of points to be controlled. A reconnaissance and monumentation party can determine which points are intervisible and, from this, which are the "possible observations". It will be assumed that if stations are intervisible, directions, distances and azimuths are all "possible observations" from each of the intervisible stations to the other.

The uncertainties in determination of the positions of net control points are solved once the observations themselves are performed and adjusted. With a knowledge of the accuracies of observations proposed to establish the network, an idea of the uncertainties, however, can be obtained from a knowledge of the approximate locations alone. It is upon this first approximate determination of the uncertainties that the design will be based. This idea is not new and, for example, was used

in "design of experiments" type problems for analysis of variances by LaRue (1964), Richards (1961) among others.

The agreement between the final estimate of uncertainty and this first approximation is, of course, dependent in part upon the quality of the values for approximate coordinates. Agreement in variances to one or two figures does not seem to be unreasonable when approximate coordinates are obtained from medium scale maps (LaRue, 1964).

1.4 Concept of the method of solution of observations.

Utilizing the approximate values for the positions of stations and a knowledge of user requirements, this study suggests the construction of a variance/covariance (V/C) matrix of parameters based solely on user requirements. This matrix will then be used in determination of which observations will be required. A solution for the elements in the variance/covariance matrix mentioned above will be formulated in Chapter 4. These elements are considered as unknowns and the reader is advised to differentiate between them and the station locations, which are also unknown and cannot be solved for until the design is made and the selected observations performed. Two alternate methods of selection of observations will be discussed based, in part, upon the above variance/covariance matrix, in Chapters 5 and 6.

Properly one might ask why this entire study is necessary. If a reconnaissance is available and approximate coordinates for points in the net are known, a minimal network of observations can be formed, solved and first approximations to coordinate uncertainties and uncertainties of derived quantities can be computed. In those areas

where user requirements are not satisfied, additional "observations" may be iteratively added and uncertainties recomputed until, by trial and error, a final design is arrived at. This is certainly possible and feasible but has some severe drawbacks. These drawbacks are:

1. This "trial and error" method is very dependent upon the experience of the designer and, to some extent, his fortunate choice of observations. While it may work on the first or second iteration, there is a possibility that it will take many trials and this may be unsatisfactory.
2. Usually in the "trial and error" approach, even if performed station by station, observations are added but seldom deleted. This solution may therefore contain observations which impart very little new information, but which add cost. If observations are added and subtracted, either intuitively or systematically, the possiblity of a fairly rapid determination of a usable design decreases. This is particularly true as the number of points to be controlled increases.

The objective is then the systematic and partially automated solution to the choice of observations which take advantage of but do not depend upon the designer's expertise. This solution will be applicable even in the case of unusual derived quantities and unusual observation types where expertise does not yet exist.

To illustrate each phase in the design, examples involving two test nets will approximate coordinates and lists of possible observations are used. Tables 1.1 through 1.4 and Figures 1.1 and 1.2 describe

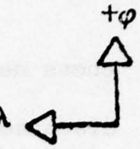
these nets. Variances are assigned to each "possible observation" and discussed in Chapter 5. Unfortunately, small, compact examples in horizontal control networks do not illustrate the concepts presented.

The examples given involve many tables and were included not only to illustrate concepts but also to provide some idea of the practical problems which will be encountered in implementing the methods. Originally, 4 test nets were generated, varying in size from 4 to 14 stations. Of these, only one was an idealized configuration, while the others ranged from strong but physically realizable to very weak configurations. The methods given in succeeding chapters worked equally well on all nets.

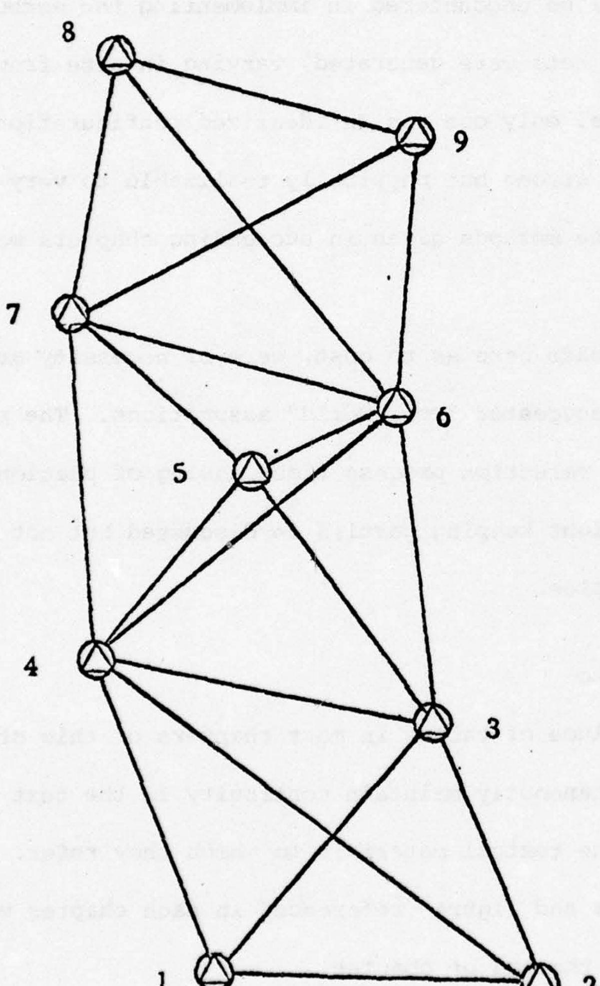
Suppositions made here as to cost, were of necessity arbitrary and do not constitute suggested "real world" assumptions. The problem of integrating in the selection process time-phasing of station occupation by observing and light keeping parties is discussed but not implemented in any of the examples.

1.5 Procedural Note

Due to the volume of tables in most chapters of this study, it was difficult to simultaneously maintain continuity in the text and have tables following the textual materials to which they refer. For that reason, most tables and figures referenced in each chapter were placed in serial order at the end of chapter.



<u>NETWORK T3</u> - **sketch of station locations**



Stations (△) joined by solid lines indicate that they are intervisible

Figure 1.1

LIST OF POSITIONS IN NET
(APPROXIMATE)

NOTE - ALL LATITUDES POSITIVE NORTHWARD
ALL LONGITUDES POSITIVE WESTWARD

NUMBER STATION	DEGREES	MINUTES LATITUDE	SECONDS	DEGREES	MINUTES LONGITUDE	SECONDS
1	40	10	36.128	102	49	2.374
2	40	10	11.933	102	37	17.130
3	40	17	19.529	102	40	50.317
4	40	19	24.745	102	53	9.628
5	40	24	26.471	102	47	19.842
6	40	26	11.942	102	42	6.084
7	40	28	41.891	102	53	38.966
8	40	35	56.165	102	51	57.423
9	40	33	40.124	102	41	10.364

Table 1.1

DESCRIPTION OF THE OBSERVATIONS

KEY - D INDICATES A DIRECTION
(UNITS FOR VARIANCE ARE ARC SECONDS SQUARED)

A INDICATES AN AZIMUTH
(UNITS FOR VARIANCE ARE ARC SECONDS SQUARED)

S INDICATES A DISTANCE
(UNITS FOR VARIANCE ARE METERS SQUARED)

OBSERVATION NUMBER	TYPE	A PRIORI VARIANCE	FROM STATION	TO STATION
1	D	0.160	1	2
2	D	0.160	1	3
3	D	0.160	1	4
4	D	0.160	2	1
5	D	0.160	2	3
6	D	0.160	2	4
7	D	0.160	3	1
8	D	0.160	3	2
9	D	0.160	3	4
10	D	0.160	3	5
11	D	0.160	3	6
12	D	0.160	4	1
13	D	0.160	4	2
14	D	0.160	4	3
15	D	0.160	4	5
16	D	0.160	4	6
17	D	0.160	4	7
18	D	0.160	5	3
19	D	0.160	5	4
20	D	0.160	5	6
21	D	0.160	5	7
22	D	0.160	6	3
23	D	0.160	6	4
24	D	0.160	6	5
25	D	0.160	6	7
26	D	0.160	6	8
27	D	0.160	6	9
28	D	0.160	7	4
29	D	0.160	7	5
30	D	0.160	7	6
31	D	0.160	7	8
32	D	0.160	7	9
33	D	0.160	8	6
34	D	0.160	8	7
35	D	0.160	8	9

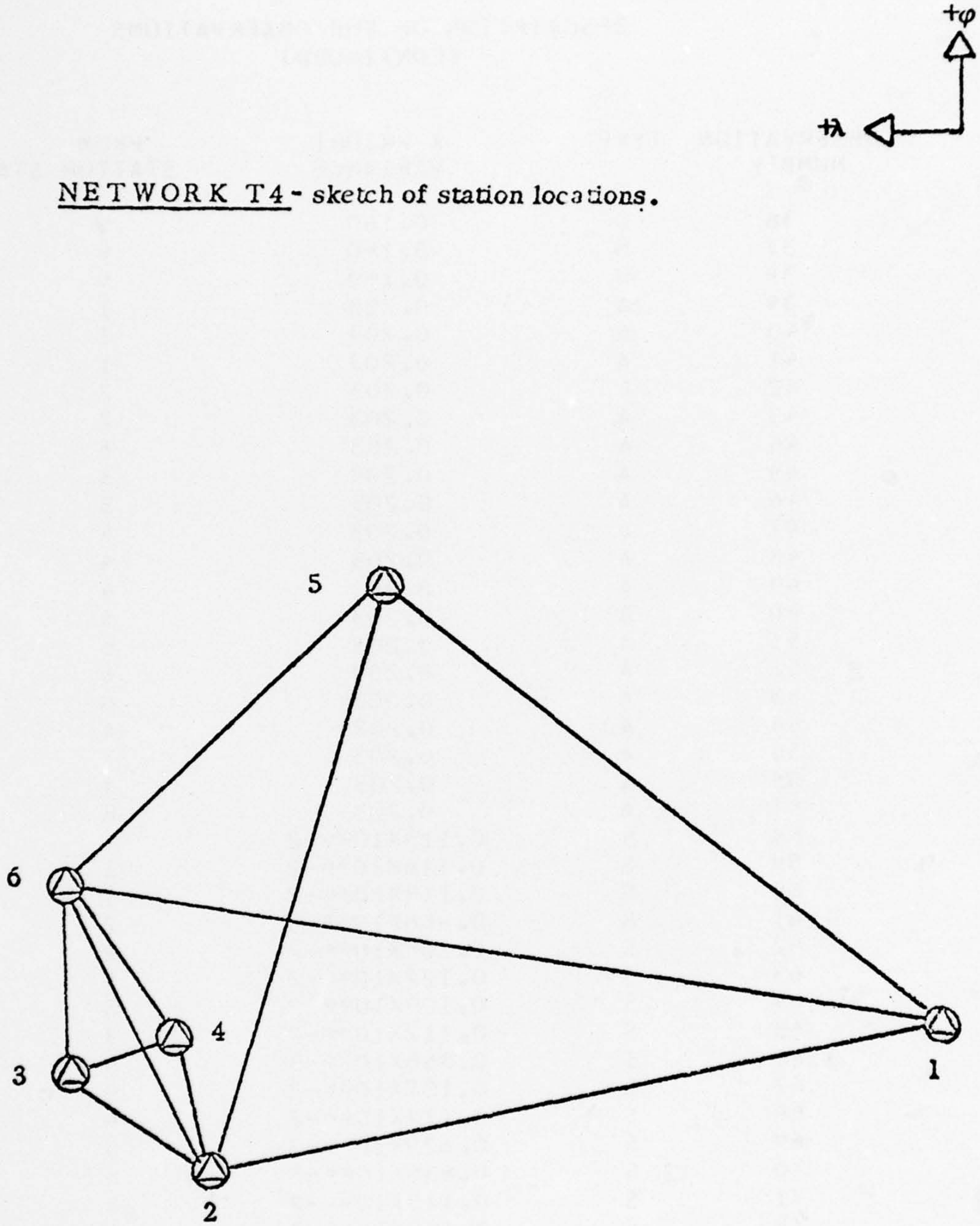
Table 1.2

DESCRIPTION OF THE OBSERVATIONS
(CONTINUED)

OBSERVATION NUMBER	TYPE	A PRIORI VARIANCE	FROM STATION	TO STATION
36	D	0.160	9	6
37	D	0.160	9	7
38	D	0.160	9	8
39	A	0.203	1	2
40	A	0.203	1	3
41	A	0.203	1	4
42	A	0.203	2	3
43	A	0.203	2	4
44	A	0.203	3	4
45	A	0.203	3	5
46	A	0.203	3	6
47	A	0.203	4	5
48	A	0.203	4	6
49	A	0.203	4	7
50	A	0.203	5	6
51	A	0.203	5	7
52	A	0.203	6	7
53	A	0.203	6	8
54	A	0.203	6	9
55	A	0.203	7	8
56	A	0.203	7	9
57	A	0.203	8	9
58	S	0.113X10**-2	1	2
59	S	0.116X10**-2	1	3
60	S	0.118X10**-2	1	4
61	S	0.968X10**-3	2	3
62	S	0.205X10**-2	2	4
63	S	0.122X10**-2	3	4
64	S	0.109X10**-2	3	5
65	S	0.112X10**-2	3	6
66	S	0.866X10**-3	4	5
67	S	0.137X10**-2	4	6
68	S	0.117X10**-2	4	7
69	S	0.629X10**-3	5	6
70	S	0.836X10**-3	5	7
71	S	0.115X10**-2	6	7
72	S	0.158X10**-2	6	8
73	S	0.954X10**-3	6	9
74	S	0.937X10**-3	7	8
75	S	0.136X10**-2	7	9
76	S	0.107X10**-2	8	9

Table 1.2 (Continued)

NETWORK T4 - sketch of station locations.



Stations (△) joined by solid lines indicate that they are intervisible

Figure 1.2

LIST OF POSITIONS IN NET
(APPROXIMATE)

NOTE - ALL LATITUDES POSITIVE NORTHWARD
ALL LONGITUDES POSITIVE WESTWARD

NUMBER STATION	DEGREES	MINUTES LATITUDE	SECONDS	DEGREES	MINUTES LONGITUDE	SECONDS
1	38	52	44.200	97	00	22.000
2	38	48	5.400	97	30	22.800
3	38	50	59.700	97	36	2.200
4	38	52	9.400	97	31	50.600
5	39	6	23.500	97	23	2.900
6	38	57	5.700	97	36	13.200

Table 1.3

DESCRIPTION OF THE OBSERVATIONS

KEY - D INDICATES A DIRECTION
(UNITS FOR VARIANCE ARE ARC SECONDS SQUARED)

A INDICATES AN AZIMUTH
(UNITS FOR VARIANCE ARE ARC SECONDS SQUARED)

S INDICATES A DISTANCE
(UNITS FOR VARIANCE ARE METERS SQUARED)

OBSERVATION NUMBER	TYPE	A PRIORI VARIANCE	FROM STATION	TO STATION
1	D	0.320	1	2
2	D	0.320	1	5
3	D	0.320	1	6
4	D	0.320	2	1
5	D	0.320	2	3
6	D	0.320	2	4
7	D	0.320	2	5
8	D	0.320	2	6
9	D	0.320	3	2
10	D	0.320	3	4
11	D	0.320	3	6
12	D	0.320	2	4
13	D	0.320	4	3
14	D	0.320	4	6
15	D	0.320	5	1
16	D	0.320	5	2
17	D	0.320	5	6
18	D	0.320	6	1
19	D	0.320	6	2
20	D	0.320	6	3
21	D	0.320	6	4
22	D	0.320	6	5
23	S	0.375X10**-2	1	2
24	S	0.340X10**-2	1	5
25	S	0.482X10**-2	1	6
26	S	0.718X10**-3	2	3
27	S	0.616X10**-3	2	4
28	S	0.275X10**-2	2	5
29	S	0.127X10**-2	2	6
30	S	0.549X10**-3	3	4
31	S	0.800X10**-3	3	6
32	S	0.790X10**-3	4	6
33	S	0.182X10**-2	5	6

Table 1.4

DESCRIPTION OF THE OBSERVATIONS
(CONTINUED)

OBSERVATION NUMBER	TYPE	A PRIORI VARIANCE	FROM	TO
34	A	0.203	1	2
35	A	0.203	1	5
36	A	0.203	1	6
37	A	0.203	2	1
38	A	0.203	2	3
39	A	0.203	2	4
40	A	0.203	2	5
41	A	0.203	2	6
42	A	0.203	3	2
43	A	0.203	3	4
44	A	0.203	3	6
45	A	0.203	4	2
46	A	0.203	4	3
47	A	0.203	4	6
48	A	0.203	5	1
49	A	0.203	5	2
50	A	0.203	5	6
51	A	0.203	6	1
52	A	0.203	6	2
53	A	0.203	6	3
54	A	0.203	6	4
55	A	0.203	6	5

Table 1.4 (Continued)

Chapter 2. Terminology and Notation

2.1 Introduction.

The purpose of this chapter is to define the basic concepts which will be applied in later work. Although specifically to be applied to the design of horizontal control, the basic ideas developed in this chapter will be applicable to any "design of experiments" problem. The discussion of mathematical models, variance and propagation of errors should not be considered as applied only to the horizontal control itself. These concepts will also be correct for the definition of the off-diagonal terms of a hypothetical variance-covariance matrix in Chapter 4 and a solution to the recovery of weights problem derived in Chapter 6.

Consider the mathematical model

$$0 = f(X_a, L_a) = L_a - F(X_a)$$

relating a set of quantities, L_a , which can be the theoretical (true) or adjusted values of observations in the case of horizontal control establishment, through a set of parameters, X_a . If the number of quantities L_a is n and the number of parameters is u , when $n > u$ the standard minimum variance solution for the corrections $\hat{X}$ to some approximate values of parameters, X_o , is:

$$\hat{X} = -(A'\Sigma^{-1}A)^{-1}A'\Sigma^{-1}W$$
$$\mathcal{E}(X_o + \hat{X}) = X_{TRUE} \qquad (2.1)$$

where $A \equiv \frac{\partial f}{\partial X}$, the design matrix of parameters, is evaluated at L_b, X_o (L_b is a vector of "observations" which has associated with it a variance-covariance matrix, Σ). The vector W is called the misclosure, $f(X_o, L_b)$, or $L_b - F(X_o)$ and X is the true value of parameters.

The matrix $N \equiv A'\Sigma^{-1}A$, will be referred to as the network free normals or free normal matrix which in every case will be rank deficient (i.e., Rank $(N) < u$).

The estimate of the variance-covariance (V/C) matrix for the parameters, Σ_X, is:

$$\Sigma_X = (A'\Sigma^{-1}A)^{-1} = N^{-1} \tag{2.2}$$

Often in geodetic problems, however, N is a positive semidefinite matrix (i.e., Rank $(N) < u$). This does not mean that no solution exists. On the contrary, many solutions, $X_o + \hat{X}$, satisfy the minimization condition, none of which is unique, in the sense that many different values of $X_o + \hat{X}$ satisfy the mathematical model (Graybill, (1961). Customarily, a constraint is applied, either absolutely or with an appropriate weight, as indicated in Pope (1971).

Weighted constraints may be thought of as the additional contribution to the normal equations of additional design matrix rows from direct observations of some (or all) of the parameters. Thus equation (2.1) becomes:

$$\hat{X} = -(A'\Sigma^{-1}A + P_x)^{-1}A'\Sigma^{-1}W \tag{2.3}$$

where P_x is a diagonal matrix whose non-zero elements are the weights assigned to specific parameters. Note that in this form the "observed" parameters are assumed to have the same values as those used as (2.1) above, X_o .

The inherent uncertainties introduced to the parameters by the "observations" themselves are among those items to be studied, due to the fact that the usual minimum constraint solutions, which, in effect, fix one or more of the parameters so that the rank and order of the normal matrix are equal but no more than that number required to accomplish this equalization of rank and order, are unsatisfactory. Since, a priori, no station is known exceptionally well as compared to any other in the net as a result of measurements taken within the net, fixing one or more of the parameters distorts the V/C matrix of the remaining parameters. This distortion resulting from fixing stations takes the form of small error ellipses for station coordinates in the immediate vicinity of fixed stations. The area of the ellipse tends to increase for stations at greater distances from the fixed stations. This pattern of increasing areas of error ellipses is not easily reconcilled physically with the quality of the observations which define the stations.

A constraint which does not cause this kind of distortion is that implied by the generalized inverse family of solutions to the <u>consistent</u> set of normal equations, (2.1), as described in Uotila (1974b) and Graybill (1969). Since this family also presents problems with uniqueness, a unique member of the generalized inverse group is chosen, hereafter called the pseudo-inverse and symbolized as N^+. A matrix

fulfilling the following four conditions (2.4-1-4) defines a unique solution to eq (2.1) and as indicated in a later section, a unique V/C matrix of parameters.

$$N N^{+} N = N \qquad 2.4\text{-}1$$

$$N^{+} N N^{+} = N^{+} \qquad 2.4\text{-}2$$

$$(N N^{+})' = N N^{+} \qquad 2.4\text{-}3$$

$$(N^{+} N)' = N^{+} N \qquad 2.4\text{-}4$$

The constraint implied by the pseudo-inverse is the minimization of $X'X$, subsequent to the minimization of the weighted sum of the squares of the residuals of the observations. The physical implications of the constraint as well as an alternate view of the solution vector, in line with the thinking of Grafarend (1973) will be discussed in section 2.3.2.

To demonstrate that the pseudo-inverse solution does minimize $X'X$, define a function ψ as indicated below and minimize it with respect to the only variable, X.

$$\underset{1,1}{\psi} = \underset{1u}{X'}\,\underset{u1}{X} - 2\,\underset{1u}{J'}\,(\underset{uu}{N}\,\underset{u1}{X} + \underset{u1}{U})$$

where

$$\underset{u1}{U} \equiv \underset{un}{A'}\,\underset{nn}{\Sigma^{-1}}\,\underset{n1}{W}$$

and J is a vector of (unknown) Lagrange multipliers.

Then $\frac{1}{2}\frac{\partial\psi}{\partial X} = X - NJ$

$$X - NJ = 0 \qquad (N \text{ symmetric}) \qquad (2.5)$$

and $$NX + \mathcal{U} = 0 \qquad \text{(original normal equations)} \tag{2.6}$$

A constraint which satisfies both (2.6) and (2.5) enforces the minimum variance and $X'X$ conditions sequentially.

Assume that for a given set of equations:

$$\underset{mm}{A}\ \underset{m1}{y} = \underset{m1}{b}$$

$y = A^{+}b$ is a solution. Does this satisfy eq (2.5) and (2.6)?

From (2.6)

$$X = -N^{+}\mathcal{U}$$

From (2.5)

$$J = N^{+}X = -N^{+}N^{+}\mathcal{U}$$

Then:

$$0 = -N^{+}\mathcal{U} + NN^{+}N^{+}\mathcal{U} \tag{2.7}$$

from (2.5) and (2.6)

but $\quad NN^{+} = (NN^{+})' = N^{+}N \quad$ from 2.4.3

and assuming symmetry in the N^{+} due to the quadratic form, N .

Substituting:

$$0 = -N^{+}\mathcal{U} + N^{+}NN^{+}\mathcal{U}$$

Since this is true for all values of $N^{+}\mathcal{U}$, eq (2.7) is satisfied.

Note that only two of the four properties of the pseudo-inverse are required to prove the proposition that a minimum $X'X$ constraint solution is attained. These properties are 2.4.2 and 2.4.3

Alternatively, if equation 2.7 is again considered

$$0 = -N^{+}\mathcal{U} + NN^{+}N^{+}\mathcal{U} \tag{2.7}$$

it will now be shown to be true if properties 2.4.1 and 2.4.4 are true. Substituting the result of the first minimization in equation 2.7, if $NX + U = O$ then $U = -NX$ and $N^{T}NX - \underline{NN^{+}}N^{T}NX = O$

But $NN^{+} = (NN^{+})' = N^{+}N$ using property 2.4.4, so equation 2.7 becomes

$$N^{+}NX - N^{+}\underline{NN^{+}N}X = O$$

Now $NN^{+}N = N$ using property 2.4.1, so the above further reduces to $N^{+}NX - N^{+}NX = O$

where $NX \neq O$ if $U \neq O$. That is, if the U vector is not identically zero, this implies that some correction to the parameters exist, i.e. $X \neq O$.

Therefore, either 2.4.1 and 2.4.4 or 2.4.2 and 2.4.3 are required to prove the minimum $X'X$ condition. If the usual approach is taken to the definition of generalized inverses, the least restrictive of the family obeys at least 2.4.1 and so the former of the two conditions is less restrictive than the latter. Also, since N is a quadratic form (and therefore symmetric) and N^{+} is a variance covariance matrix and must be symmetric, property 2.4.3 implies property 2.4.4. It is unclear why the above approach to generalized matrices is taken, and in the following development, neither the assumption that an inverse obeying 2.4.2 must obey also 2.4.1, nor the implication that 2.4.3 implies 2.4.4 is enforced.

Therefore, any matrix, G, which has the properties:

$$GNG = G$$

$$(NG)' = NG$$

where $\hat{X} = -G\,U$

or, alternatively, any matrix, H , which has the properties:

$$NHN = N$$

$$(HN)' = HN$$

where $\hat{X} = -H\,U$

lead to the <u>same</u> solution vector, $\hat{X}$, which is the minimum $X'X$ or minimum norm solution.

Since the same solution vector, $\hat{X}$, may come from other than pseudo-inverse solutions, it is appropriate to ask why the pseudo-inverse rather than G or H is used in this study. There are three principle reasons for the choice of the pseudo-inverse. These are:

1. The pseudo-inverse is unique and will be used to form a unique variance-covariance matrix for the X parameters. While the parameter values are the same for any of the above choices of solution scheme, the uniqueness of the pseudo-inverse facilitates comparisons of solutions from different sets of mathematical models and observations.
2. The pseudo-inverse, alone in the family of generalized inverses, has the property that the pseudo-inverse of the pseudo-inverse of the free normals gives the free normals as the result. This will be demonstrated in Chapter 6 and this fact will form an integral part of the application of weight recovery techniques discussed there.

3. The pseudo-inverse is a minimum trace variance-covariance matrix for the X solution. This will be demonstrated in section 2.2 and is a consequence of the minimum norm (pseudo inverse) solution.

Since the solution vector for the parameters is now determinable, consider m quantities of interest, $\underline{Y}$, related to the parameters, $\underline{X}$, in the following manner:

$$\underset{m\,1}{\underline{Y}} = F(\underline{X})$$

Linearizing

$$\underset{m\,1}{\underline{Y}_0} + Y = \underset{m\,1}{\underline{Y}} = \underset{m\,n\,n\,1}{CX} + \underset{m\,1}{D}$$ where

$$C \equiv \frac{\partial F}{\partial \underline{Y}} \;;\; \hat{Y} = C\hat{X}$$

and

$$D \equiv \underline{Y}_0 = F(\underline{X}_0)$$

Assuming that $\mathcal{E}(\underline{X}_0 + \hat{X}) = \underline{X}$ and $\mathcal{E}(\underline{Y}_0 + \hat{Y}) = \underline{Y}$, where X is the true value of $\hat{X}$ and Y is the true value of $\hat{Y}$ and applying the definition of the variance-covariance matrix given by Graybill (1961) then:

$$\Sigma_{\underline{Y}} = \mathcal{E}(\hat{Y} + \underline{Y}_0 - (Y + \underline{Y}_0))(\hat{Y} + \underline{Y}_0 - (Y + \underline{Y}_0))' = \mathcal{E}(\hat{Y} - Y)(\hat{Y} - Y)'$$

If $\hat{Y} = C\hat{X}$ then $Y = CX$ and substituting in the definition:

$$\Sigma_{\underline{Y}} = \mathcal{E}(C\hat{X} - CX)(C\hat{X} - CX)'$$

This expression may be factored to remove the constant matrix, C, giving

$$\Sigma_{\underline{Y}} = \mathcal{E}\left[C(\hat{X} - X)(\hat{X} - X)'C'\right]$$

Since the C matrix is not affected by the expectation operator, it may be removed from the brackets.

$$\Sigma_Y = C[\mathcal{E}(\hat{X}-X)(\hat{X}-X)']C'$$

Again applying the definition of Graybill, this time for X, this becomes

$$\Sigma_Y = C[\mathcal{E}(\hat{X}+X_0-(X+X_0))(\hat{X}+X_0-(X+X_0))']C'$$
$$= C\Sigma_X C'$$

The above equation enables the planner to form an estimate of the V/C matrix of ancillary quantities from a knowledge only of the approximate values for the parameters, X_0, along with a knowledge of the variances of proposed observations.

2.2 Estimates of the variance covariance matrices of parameters.

In the case of rank difficiency in the free normal equation, the *pseudo-inverse* of the normal matrix represents the degenerate V/C matrix for the parameters (Pope (1971)). Symbolically, $\Sigma_X = N^+$, since estimates of the variances of the observations are used to form the normal matrix, i.e. $N \equiv A'\Sigma^{-1}A$. Note that this statement may be considered a consequence of the previously developed scheme for the propagation of variance, applied to the observations themselves. That is:

$$AX + W = 0 \qquad \text{(inconsistent)}$$

$$NX + U = 0 \qquad \text{(consistent through the minimization of the variance)}$$

Then $\Sigma_U = (A'\Sigma^{-1})\Sigma(\Sigma^{-1}A)$ using the rules for propagation, since $U = A'\Sigma^{-1}W$ by previous definition. Then

$$\Sigma^{-1}\Sigma\Sigma^{-1} = \Sigma^{-1}$$

Then $$\Sigma_u = A'\Sigma^{-1}A$$

If $$\hat{X} = -N^{+}u$$

Then $$\Sigma_X = -N^{+}\Sigma_u(-N^{+}) = N^{+}NN^{+} = N^{+}$$

using the characteristics of the pseudo-inverse previously stated. Pope's (1971) concept of $(A'\Sigma^{-1}A)^{+}$ as a degenerate error hyperellipsoid may be misleading. The pseudo-inverse solution of the normal equations is considered here to be the *V/C matrix of parameters* as a result of an imposed constraint just as any other minimum constraint solution is ordinarily considered. Since this is a minimum constraint solution for the adjusted observations (in the sense defined in Pope (1973)) and parameters, all the concepts of point estimation of estimable quantities apply to the N^{+} as well as to any other minimum constraint solution.

2.2.1 Estimable quantities and tests for estimability.

The definitions of estimable and linearly estimable quantities are quoted below for completeness (Graybill (1961)).

> "Definition 11.2 A parameter (a function of parameters) is said to be estimable if there exists an unbiased estimate of the parameter (of the function of the parameters).
>
> *Definition* 11.3 A *parameter* (a function of parameters) is said to be linearly estimable if there exists a linear combination of the observations whose expected value is equal to the parameters (the function of parameters), i.e., if there exists an unbiased estimate."

Mathematically, for any function of parameters:

$$\underline{Y} = F(\underline{X})$$

$$\delta \underline{Y} = C \delta \underline{X}$$

$$C = \frac{\partial F}{\partial \underline{X}} \quad \text{(linearized)}, \quad \underline{Y} = F(\underline{X}_0) + \delta \underline{Y}$$

and if

$$C = C N^{+} N$$

Then the quantity $\delta \underline{Y}$ is estimable with a unique, unbiased estimator, $-CN^{+}\mathfrak{U}$, and a variance $CN^{+}C'$ (Rao (1962) and Searle (1965)). The mathematical estimability criterion, above, can be used as a test of a specified quantity's estimability with respect to some set of observations and free normal equations.

In effect, since the residuals for observations for any minimum constraint solution are the same, and all linearly estimable quantities can be thought of as functions of the adjusted observed quantities, the predicted point estimate of any estimable and its variance are invariant, regardless of the choice of minimum constraint.

The consideration that the $X'X$ minimum is one of the family of minimum constraints with some physical interpretation which is then applied to the rank deficient normal equations of the free network provides a way to reason around the objections raised by Grafarend and Schaffrin (1973). This constraint, incorporated into the formation of the normal equations before an inversion is attempted, results in the elimination of some of the parameters and, with them, the rank deficiency.

The alternate to the acceptance of the constraint implied by the pseudo-inverse is to accept the Grafarend and Schaffrin argument that this V/C matrix is physically meaningless and concentrate on the variances of the estimables. In subsequent sections the former approach is taken. This does not invalidate the conclusions drawn with respect to the estimables if the V/C matrix of parameters is indeed physically meaningless when derived using the pseudo-inverse. Since the working surveyor is generally not interested in non-estimable parameters, networks can be designed with the useful estimable quantities in mind and then transformed from the $X'X$ minimum into a more physically meaningful constraint system.

2.2.2 The pseudo-inverse as a minimum trace V/C matrix.

The variance covariance matrix, N^{+}, is a minimum trace V/C matrix as a consequence of the $X'X$ minimization and the use of the pseudo inverse in computations. This is shown in Pope (1971) where the trace of any other variance covariance matrix, Σ_x^D, resulting from a minimum constraint which is not the $X'X$ type, is given as

$$\operatorname{tr}\Sigma_x^D = \operatorname{tr}N^{+} + \operatorname{tr}(R'N^{+}R) = \operatorname{tr}N^{+} + \operatorname{tr}\left(S\left[(E'D)^{-1}\right]'D'N^{+}D(E'D)^{-1}S'\right)$$

where $R \equiv D(E'D)^{-1}S'$, S is an $r \times r$ matrix such that $\underset{rr}{S'}\,\underset{rr}{S} = \underset{ru}{E'}\,\underset{ur}{E}$, E is the linearization of the $X'X$ constraint and D is the linearization of the other minimum constraint. Since $\operatorname{tr} R'N^{+}R$ can be no less than zero (it is the sum of the major diagonal elements of a matrix which is of quadratic form), then $\operatorname{tr}\Sigma_x^D \geq \operatorname{tr}N^{+}$.

For the lowest value of $\operatorname{tr}\Sigma_x^D$, $D \equiv E$ and $\operatorname{tr}(R'N^{+}R)$ is zero. This is a consequence of $E'N^{+}E$ being zero, which can be seen if properties 2.4.2 and 2.4.3 are applied to that term. If as a definition, the matrix must fulfill these conditions:

$$AE = 0 \quad \text{and} \quad \det[E'E] \neq 0$$

Then $E'\underline{N}^{+}E = E'N^{+}\underline{NN^{+}}E = E'N^{+}N^{+}\underline{NE} = E'N^{+}N^{+}A'\Sigma^{-1}\underline{AE} = 0$

The constraint implied by the $X'X$ minimization will be discussed in further detail in section 2.3.2. It should be noted here, however, that this minimum trace condition applies to the $X'X$ condition as compared to others in the minimum constraint family only, not to overconstraints.

2.3 Application to horizontal control networks.

2.3.1 The mathematical model.

This study will consider the problem of designing the net of observations required to position points on a datum, specifically the present North American Datum. Three types of observations will be used in this design. They are not the exclusive choices, but represent the most common ones used presently in establishing horizontal control. These types are:

1. Directions as observed on the datum.
2. Distances as measured on the datum.
3. Astronomic azimuths transformed into Laplace azimuths in the usual way (Rapp (1969)) as measured on the datum.

The reduction of the actual observations to this datum will not be considered. Estimates of variances for observations will be assumed to refer to these datum observed quantities.

There are many versions of the coefficients in the linearization of the mathematical model, eq (2.1), in this case. These range from a planar approximation for very local nets to equations developed for very widely separated points. Arbitrarily, the linearized equations and coefficients corresponding to a latitude-longitude system developed by

Rapp (1969) and the Gaussian Mid-Latitude formulas (Jordan, Eggert, Kneissl, 1959) as stated by Gergen (1970) are used in this presentation. These, following the usual U.S. custom, measure azimuths clockwise from the south and longitudes positively to the west. These equations are given below.

1. For a direction, D_{ik} , observed at station i to station k

$$\delta D_{ik} = \delta z_i - \frac{M_i}{S_{ik}} \sin A_{ik}\, \delta\phi_i - \frac{M_k}{S_{ik}} \sin A_{ki}\, \delta\phi_k - \frac{N_k}{S_{ik}} \cos\phi_k\, \delta\lambda_k + \frac{N_k}{S_{ik}} \cos\phi_k\, \delta\lambda_i \tag{2.10}$$

2. For a distance, S_{ik} , observed between stations i and k

$$\delta S_{ik} = M_i \cos A_{ik}\, \delta\phi_i + M_k \cos A_{ki}\, \delta\phi_k - N_k \cos\phi_k \sin A_{ki}\, \delta\lambda_k + N_k \cos\phi_k \sin A_{ki}\, \delta\lambda_i \tag{2.11}$$

3. For an azimuth, A^*_{ik} , observed at station i from the south to station k.

$$\delta A^*_{ik} = -\Big[\frac{M_i}{S_{ik}} \sin A_{ik}\, \delta\phi_i + \frac{M_k}{S_{ik}} \sin A_{ki}\, \delta\phi_k - \frac{(N_k \cos\phi_k + S_{ik} \sin\phi_i)}{S_{ik}}\, \delta\lambda_i + N_k \cos\phi_k \cos A_{ki}\, \delta\lambda_k\Big] \tag{2.12}$$

The equations used in solving the direct and inverse problems for stations in the net are:

$$-S \sin A = N \Delta\lambda \cos\phi \left[1 - \frac{1}{24}\Delta\lambda^2 \sin^2\phi + \frac{1+\eta^2-9\eta^2 t^2}{24 V^4}\Delta\phi^2\right]$$

$$-S \cos A = M \Delta\phi \cos\frac{\Delta\lambda}{2}\left[1 + \frac{1-2\eta^2}{24}\Delta\lambda^2 \cos^2\phi - \frac{\eta^2(t^2-1-\eta^2-4\eta^2 t^2)}{8 V^4}\Delta\phi^2\right]$$

$$\Delta A = \Delta\lambda \sin\phi \left[1 + \frac{1+\eta^2}{12}\Delta\lambda^2 \cos^2\phi + \frac{3+8\eta^2+5\eta^4}{24 V^4}\Delta\phi^2\right]$$

Where M_j , N_j are radii of the reference datum ellipsoid in meridian and prime vertical respectively, at station j (if unsubscripted they refer to the mean latitude between two stations)

$$\Delta\phi = \phi_k - \phi_i$$
$$\Delta\lambda = \lambda_k - \lambda_i$$
$$\eta^2 = e'^2 \cos^2\phi$$
$$t = \tan\phi$$
$$V^2 = 1 + \eta^2$$

A_{ik} is the azimuth, measured from the south, to station k from station i.

A, η, t, V, ϕ correspond to the mean latitude.

$$\Delta A = A_{ik} - A_{ki} + \pi$$

and e' is the second eccentricity.

2.3.2 Some comments on the physical interpretation of the minimum constraint.

Three types of networks will be discussed in this study. These are:

1. Networks formed from directions, azimuths and distances (rank deficient by two)
2. Networks formed from directions and distances (rank deficient by three)
3. Networks formed from directions and azimuths (rank deficient by three).

The above were chosen from all the possible combinations of directions, distances and azimuths, as representing the most commonly observed types.

2.3.2.1 Networks with directions, azimuths and distances.

The minimum constraint for this formation of the normal equations is the definition of an origin, classically done by fixing the latitude and longitude of one station. This is not the only choice of constraint in this situation. The fixing of any individual latitude and longitude of different points within the net will also result in a minimum constraint. As indicated in Pope (1973), the pseudo-inverse in this case corresponds to the Cayley inverse of the following bordered matrix:

$$\begin{bmatrix} \underset{u\;u}{A'\Sigma^{-1}A} & \underset{u\;r}{E} \\ \underset{r\;u}{E'} & \underset{r\;r}{\bar{O}} \end{bmatrix} \qquad r \equiv u - \mathrm{Rank}(A'\Sigma^{-1}A)$$

where E, the null space matrix of the free normal equation matrix, N, (for formal definition, see Graybill (1969), DFN 5.4.2) is a column matrix which is u by r. Any matrix such that

$$NE = O$$

or

$$AE = O$$

and determinant $(E'E) \neq O$

define either the null space matrix or a linear combination of the columns of the null space matrix. These may be attributed to an absolute constraint $G(\underline{X}) = O$ which, in linearized form, becomes:

$$\underset{r\,u\,u\,1}{E'X} + \underset{r\;1}{W_E} = O \qquad \text{where } W_E \equiv G(\underline{X}_o)$$

It would be convenient to think of this as an inner origin constraint in the sense defined for space rectangular coordinates by Blaha (1971)

and Pope (1973), where the physical constraint implies the fixing to some value of the sum of the corrections to each of the independent coordinates. Thus, for n points, the adjusted coordinate is the approximate coordinate with the addition of the difference between the fixed value and the sum of all other corrections, to like coordinates. Examination reveals that this is not the case in the ϕ , λ , z coordinate system, which is not a mathematically "flat space".

Consider the constraints:

$$\sum_{i=1}^{n} \delta\phi_i \quad = \text{constant}$$

$$\sum_{i=1}^{n} \delta\lambda_i \quad = \text{constant}$$

which linearize to:

$$E'_\phi = [0 \,\vdots\, 1 \,\vdots\, 0 \,\vdots\, \cdots\cdots\cdots \,\vdots\, 0 \,\vdots\, 1 \,\vdots\, 0] \tag{2.14}$$

$$E'_\lambda = [0 \,\vdots\, 0 \,\vdots\, 1 \,\vdots\, \cdots\cdots\cdots \,\vdots\, 0 \,\vdots\, 0 \,\vdots\, 1] \tag{2.15}$$

where

$$x' = [\delta z_1 \,\vdots\, \delta\phi_1 \,\vdots\, \delta\lambda_1 \cdots \,\vdots\, \delta z_i \,\vdots\, \delta\phi_i \,\vdots\, \delta\lambda_i \,\vdots \cdots \vdots\, \delta z_n \,\vdots\, \delta\phi_n \,\vdots\, \delta\lambda_n]$$

For the latitude, this implies that for a direction i to k,

$$-\frac{M_i}{S_{ik}} \sin A_{ik} - \frac{M_k}{S_{ik}} \sin A_{ki} \stackrel{?}{=} 0$$

from the formation of the product AE using the coefficient in eq (2.14).

In a very local area

$$\sin A_{ik} = \sin A_{ki}$$

$$M_i = M_k$$

but in general, this is not correct for the distance involved in the test nets used here.

For the longitude unknowns, this constraint, the vector E_λ, equation (2.15), satisfies the observations of direction and distance, but when applied to those for azimuth it too falls, even in very local areas because of the $\sin\phi_i$ term in the coefficient for station i (the observing station) of the linearized azimuth observation equation (2.12). This extra term, not present in the direction equations, is introduced to account for a change in the azimuth due to a change in the approximate correction made to the astronomic azimuth for the deflection of the vertical in the longitudinal direction (Rapp (1969)).

Whatever the exact form of the E constraint, however, an origin of some sort is defined. In further discussion, this will be referred to for convenience as the inner origin, but its definition will not be that of Blaha and Pope. For test nets of small size (up to 15 stations) with distances not exceeded 100 km at moderate latitudes, generally, the following quantities are estimable:

1. Distances between stations in the net,
2. Azimuths from station i to j ($i \neq j$)
3. Angles made by lines of sight from the k and j stations as observed from i ($i \neq j \neq k$)

The above is not an all inclusive list and testing for estimability for other quantities will be discussed in section 2.4.

2.3.2.2 Networks with directions and distances.

In this situation, the design matrix is

$$A = \begin{bmatrix} A_{DIR} \\ A_{DIST} \end{bmatrix}$$

where X has the same parameter order as in 2.3.2.1.

Surprisingly, the simple inner origin condition for longitude is now usable, indicating an origin change from the previous network, contrary to the scheme suggested by Pope (1973), for space rectangular coordinate systems. Whether this is due to the very different coordinate systems considered here or to the reduction of the astronomic azimuth is unclear.

The rotation null space, E_r, for the cartesian coordinates given in Pope (1973) is also usable only in a very limited sense. If

$$E_r' = [1 \vdots -\lambda_1 \vdots \phi_1 \vdots \cdots\cdots \vdots 1 \vdots \lambda_n \vdots \phi_n]$$

for this datum and for a direction

$$1 - \frac{M_i}{S_{ik}} \sin A_{ik} \lambda_i - \frac{M_k}{S_{ik}} \sin A_{ki} \lambda_k - \frac{N_k}{S_{ik}} \cos \phi_k \cos A_{ki} \phi_i + \frac{N_k}{S_{ik}} \cos \phi_k \cos A_{ki} \phi_k \overset{?}{=} 0$$

which is true in very local areas.

Again, while the planar solutions for the E constraint are apparently in only very limited value in physical interpretation, the following are estimable quantities useful to surveyors.

1. Distances between points in the net.

2. Angles involving net points as described in 2.3.2.1.

2.3.2.3 Networks with directions and azimuths.

The same type of analysis can be performed on this type of design matrix. Again the physical interpretation of the implied constraint is unclear, but the following quantities are estimable:

1. Azimuths between points in the net.
2. Angles involving net points as previously defined.

2.3.2.4 Interpretation of the variance derived from the pseudo-inverse.

Whatever the physical interpretation of implied constraints, it is convenient to consider the Σ_x matrix defined by the pseudo-inverse to be a measure of the error implied in station coordinates due strictly to the uncertainty in the observations. As an empirical rule of thumb resulting from numerical tests on a series of networks with different configurations, which will provide some starting point in the investigation of the variances for different designs, the standard deviation of a station coordinate computed through some other minimum constraint will be expected to vary from that of the pseudo-inverse constraint by up to three to five times the magnitude of the pseudo solution. This will be useful in its reverse application where some maximum standard deviation for any station in the net is specified for a given (or unstated) minimum constraint, other than the pseudo-inverse. For those adopting the view that the pseudo-inverse V/C matrix of parameters is meaningless, the numerical operation is still correct in the investigation of those quantities which are estimable, even though the interpretation is unacceptable when discussing the variance of station coordinates.

2.4 Other user specified estimable quantities.

If doubt exists as to what is estimable for a specific circumstance, and application of the word definition given by Graybill is unconvincing, the definition of Rao may be applied. This definition is exact for an exactly defined linear model. In the case of horizontal control, where the linearized model is used and coefficients are derived making assumptions and approximations the definition may be less clear. To check estimability, it is suggested that a sample solution be performed using at least one of each type of observation thought to be needed with at least one degree of freedom. This will define N and $N^{\dagger}$. To test the effect of approximations and numerical roundoff, not only should the user required quantity in question be tested, but also some of the observations used in the actual formation of N should also be tested using the mathematical definition in section 2.2.1. The C (see section 2.2.1) matrix (vector) in this instance should then contain rows from the original design matrix. The test of some rows of the design matrix will be a very good indicator of the extent of roundoff present and can be used as a yardstick by which to judge the estimability of the user required quantity in question.

Two numerical examples of this type of estimability testing are given here as illustrations. The estimables to be tested are derivable azimuths in the first test and distances in the second. In tables 2.2 and 2.3, these estimables, in linearized form are denoted as members of the design matrix of observation, A, since they are "possible observations" for network T3 as well as estimables (C). Firstly, network T3 is formed with directions and distances (rank deficient by three) and the pseudo-

inverse performed upon the free network normal equations. Then, the matrix of some representative direction, distance and azimuth observations is formed, Table 2.1, and tested in the manner described in section 2.2.1. Table 2.2 indicates the result of this testing. Note clear difference in magnitude between the roundoff induced "noise" for coefficients of rows (observations, see Table 1.2) 37, 38 (directions) and rows 41, 42 (distances) and the coefficients in rows 39 and 40 which are azimuths (and theoretically non-estimable from this network).

The network was reformed with observations of directions and azimuths used and a matrix of directions, azimuths and distances tested. Table 2.3 is the result of that test. Note again the clear differences in magnitudes of coefficients between the roundoff present for rows 37, 38, 39 and 40 (directions and azimuths respectively) and the coefficients of rows 41 and 42 (the non-estimable distances).

These examples illustrate in a clear way the manner in which more complex user proposed estimables may be checked against designer choice of observation types. In tests performed on such other estimables as areas and angles, for the four test networks mentioned in section 1.4, this pattern of small magnitude roundoff noise in $CN^{+}N$ coefficients, as compared to those of the C matrix, for estimable quantities as opposed to distinctly larger magnitude coefficients in $CN^{+}N$ for non-estimable quantities was always evident. In conclusion, it would appear that the estimability of any user specified quantity can always be ascertained by testing in the manner suggested in section 2.2.1 and illustrated above.

DESCRIPTION OF THE OBSERVATIONS

KEY - D INDICATES A DIRECTION
(UNITS FOR VARIANCE ARE ARC SECONDS SQUARED)
A INDICATES AN AZIMUTH
(UNITS FOR VARIANCE ARE ARC SECONDS SQUARED)
S INDICATES A DISTANCE
(UNITS FOR VARIANCE ARE METERS SQUARED)

OBSERVATION NUMBER	TYPE	A PRIORI VARIANCE	FROM STATION	TO STATION
1	D	0.160	1	2
2	D	0.160	1	3
3	D	0.160	1	4
4	D	0.160	2	1
5	D	0.160	2	3
6	D	0.160	2	4
7	D	0.160	3	1
8	D	0.160	3	2
9	D	0.160	3	4
10	D	0.160	3	5
11	D	0.160	3	6
12	D	0.160	4	1
13	D	0.160	4	2
14	D	0.160	4	3
15	D	0.160	4	5
16	D	0.160	4	6
17	D	0.160	4	7
18	D	0.160	5	3
19	D	0.160	5	4
20	D	0.160	5	6
21	D	0.160	5	7
22	D	0.160	6	3
23	D	0.160	6	4
24	D	0.160	6	5
25	D	0.160	6	7
26	D	0.160	6	8
27	D	0.160	6	9
28	D	0.160	7	4
29	D	0.160	7	5
30	D	0.160	7	6
31	D	0.160	7	8
32	D	0.160	7	9
33	D	0.160	8	6
34	D	0.160	8	7
35	D	0.160	8	9

Table 2.1

DESCRIPTION OF THE OBSERVATIONS
(CONTINUED)

OBSERVATION NUMBER	TYPE	A PRIORI VARIANCE	FROM STATION	TO STATION
36	D	0.160	9	6
37	D	0.160	9	7
38	D	0.160	9	8
39	A	0.203	1	2
40	A	0.203	8	9
41	S	0.113X10**-2	1	2
42	S	0.107X10**-2	8	9

Table 2.1 (Continued)

CHECK OF ESTIMABILITY

NUMBER OF OBSERVATIONS

NO. OF DIRECTIONS = 38
NO. OF AZIMUTHS = 0
NO. OF DISTANCES = 2

TOTAL NUMBER OF OBSERVATIONS = 40

Table 2.2

THE DESIGN MATRIX OF OBSERVATIONS (A)

OBSERVATION # (37)

0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	2.84D+02	1.13D+02	0.0	0.0	1.00D+00	0.0	-2.84D+02	-1.13D+02

OBSERVATION # (38)

0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	3.89D+02	1.00D+00	-8.14D+01	-3.88D+02	8.14D+01

OBSERVATION # (39)

0.0	3.81D+02	-1.27D+01	0.0	-3.80D+02	1.34D+01	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

OBSERVATION # (40)

0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	3.89D+02	0.0	-8.14D+01	-3.88D+02	8.20D+01

Table 2.2 (Continued)

THE DESIGN MATRIX OF OBSERVATIONS (A)

OBSERVATION # (41)

0.0	1.34D+00	2.36D+01	0.0	-1.41D+00	-2.36D+01	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

OBSERVATION # (42)

0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	8.17D+00	0.0	2.27D+01	-8.23D+00	-2.27D+01

Table 2.2(Continued)

THE PRODUCT OF "A*N(PSEUDO)*N"

OBSERVATION # (37)

-8.33D-09	2.10D-05	6.33D-06	-8.32D-09	1.60D-05	6.50D-06	-1.39D-08	2.20D-05	3.77D-06
-1.67D-08	3.39D-05	2.32D-06	-1.11D-08	7.71D-05	-6.36D-07	-1.67D-08	6.29D-05	-1.98D-06
-1.39D-08	2.84D+02	1.13D+02	-8.38D-09	1.96D-05	1.00D+00	-7.32D-06	-2.84D+02	-1.13D+02

OBSERVATION # (38)

2.64D-09	1.45D-05	-2.01D-06	2.65D-09	1.40D-05	-2.12D-06	4.43D-09	1.79D-05	-1.24D-06
5.32D-09	2.17D-05	-6.97D-07	3.55D-09	5.51D-05	2.10D-07	5.34D-09	5.00D-05	5.99D-07
4.44D-09	2.37D-05	1.10D-06	2.67D-09	3.89D+02	1.00D+00	-8.14D+01	-3.88D+02	8.14D+01

OBSERVATION # (39)

3.05D-02	3.78D+02	-3.60D+01	3.05D-02	-3.72D+02	-1.07D+01	5.10D-02	6.60D+00	-1.41D+01
6.12D-02	-1.07D+01	-8.43D+00	4.09D-02	-3.86D+00	2.29D+00	6.13D-02	1.43D+01	7.07D+00
5.11D-02	-1.26D+01	1.22D+01	3.07D-02	-5.17D+00	3.07D-02	2.70D+01	5.82D+00	2.06D+01

OBSERVATION # (40)

3.05D-02	-2.78D+00	-2.32D+01	3.05D-02	8.34D+00	-2.41D+01	5.10D-02	6.60D+00	-1.41D+01
6.12D-02	-1.07D+01	-8.43D+00	4.09D-02	-3.87D+00	2.29D+00	6.13D-02	1.43D+01	7.07D+00
5.11D-02	-1.26D+01	1.22D+01	3.07D-02	3.83D+02	3.07D-02	-5.44D+01	-3.83D+02	1.03D+02

Table 2.2 (Continued)

THE PRODUCT OF "A*N(PSEUDO)*N"

OBSERVATION # (41)

2.08D-10	1.34D+00	2.36D+01	2.08D-10	-1.41D+00	-2.36D+01	3.47D-10	-2.73D-06	-9.08D-08
4.17D-10	-3.68D-06	-6.14D-08	2.77D-10	-8.89D-06	1.49D-08	4.16D-10	-7.69D-06	5.24D-08
3.48D-10	-4.05D-06	7.75D-08	2.08D-10	-2.15D-06	2.08D-10	1.80D-07	-2.56D-06	1.44D-07

OBSERVATION # (42)

-2.08D-10	2.38D-06	1.59D-07	-2.08D-10	2.07D-06	1.58D-07	-3.47D-10	2.73D-06	9.09D-08
-4.17D-10	3.69D-06	6.17D-08	-2.78D-10	8.90D-06	-1.52D-08	-4.17D-10	7.70D-06	-5.23D-08
-3.48D-10	4.05D-06	-7.78D-08	-2.09D-10	8.17D+00	-2.09D-10	2.27D+01	-8.23D+00	-2.27D+01

Table 2.2 (Continued)

CHECK OF ESTIMABILITY

NUMBER OF OBSERVATIONS

NO. OF DIRECTIONS = 38
NO. OF AZIMUTHS = 2
NO. OF DISTANCES = 0

TOTAL NUMBER OF OBSERVATIONS = 40

Table 2.3

THE DESIGN MATRIX OF OBSERVATIONS (A)

OBSERVATION # (37)

0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	2.84D+02	1.13D+02	0.0	0.0	1.00D+00	0.0	-2.84D+02	-1.13D+02

OBSERVATION # (38)

0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	3.89D+02	1.00D+00	-8.14D+01	-3.88D+02	8.14D+01

OBSERVATION # (39)

0.0	3.81D+02	-1.27D+01	0.0	-3.80D+02	1.34D+01	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

OBSERVATION # (40)

0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	3.89D+02	0.0	-8.14D+01	-3.88D+02	8.20D+01

Table 2.3 (Continued)

THE DESIGN MATRIX OF OBSERVATIONS (A)

OBSERVATION # (41)

0.0	1.34D+00	2.36D+01	0.0	-1.41D+00	-2.36D+01	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

OBSERVATION # (42)

0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	8.17D+00	0.0	2.27D+01	-8.23D+00	-2.27D+01

Table 2.3 (Continued)

THE PRODUCT OF "A*N(PSEUDO)*N"

OBSERVATION # (37)

-1.31D-10	3.36D-05	-1.59D-05	-5.67D-11	3.27D-05	-7.31D-06	-1.23D-10	1.07D-05	-2.58D-05
-3.02D-10	9.34D-06	-4.54D-05	-1.54D-10	-4.75D-06	-3.59D-05	-1.74D-10	-1.27D-05	-3.07D-05
-2.70D-10	2.84D+02	1.13D+02	-1.56D-10	-2.96D-05	1.00D+00	-2.65D-05	-2.84D+02	-1.13D+02

OBSERVATION # (38)

3.65D-10	3.00D-05	4.25D-05	4.32D-10	2.95D-05	5.07D-05	6.90D-10	9.66D-06	1.25D-04
6.94D-10	8.18D-06	9.52D-05	5.05D-10	-4.33D-06	1.07D-04	8.11D-10	-1.12D-05	1.34D-04
5.71D-10	-1.14D-05	1.02D-04	3.47D-10	3.89D+02	1.00D+00	-8.14D+01	-3.88D+02	8.14D+01

OBSERVATION # (39)

3.29D-10	3.81D+02	-1.27D+01	3.46D-10	-3.80D+02	1.34D+01	5.55D-10	-3.78D-07	1.05D-04
6.79D-10	-5.28D-07	9.70D-05	4.55D-10	1.52D-07	9.88D-05	6.89D-10	6.91D-07	1.14D-04
5.81D-10	3.94D-07	1.05D-04	3.50D-10	1.16D-06	3.46D-10	5.93D-05	1.27D-06	5.26D-05

OBSERVATION # (40)

-3.40D-10	1.50D-06	-4.04D-05	-3.37D-10	1.23D-06	-4.04D-05	-5.69D-10	3.89D-07	-1.05D-04
-6.87D-10	5.23D-07	-9.70D-05	-4.59D-10	-1.51D-07	-9.88D-05	-6.85D-10	-6.95D-07	-1.14D-04
-5.73D-10	-3.90D-07	-1.05D-04	-3.46D-10	3.89D+02	-3.43D-10	-8.14D+01	-3.88D+02	8.20D+01

Table 2.3 (Continued)

THE PRODUCT OF "A*N(PSEUDO)*N"

OBSERVATION # (41)

-1.92D-06	5.75D+00	2.34D+01	7.69D-06	2.88D+00	-2.27D+01	8.00D-06	1.41D+00	1.48D+00
-1.06D-05	1.22D+00	-1.49D+00	-6.99D-07	-6.11D-01	-1.50D-01	7.57D-06	-1.64D+00	1.26D+00
-9.57D-06	-1.64D+00	-1.74D+00	-4.36D-06	-3.87D+00	4.57D-06	-7.44D-01	-3.55D+00	6.98D-01

OBSERVATION # (42)

-1.81D-06	4.16D+00	-2.14D-01	7.27D-06	4.05D+00	8.66D-01	7.57D-06	1.33D+00	1.40D+00
-1.00D-05	1.15D+00	-1.41D+00	-6.61D-07	-5.77D-01	-1.42D-01	7.16D-06	-1.55D+00	1.19D+00
-9.04D-06	-1.55D+00	-1.64D+00	-4.12D-06	4.52D+00	4.32D-06	2.20D+01	-1.16D+01	-2.20D+01

Table 2.3 (Continued)

Chapter 3. Quantification of User Requirements

3.1 Introduction.

Establishment of horizontal control at a number of locations for specific subsequent use is a matter not only of measurements but also of cost. While it would be ideal to be able to design "the best" network configuration and always observe in "the most precise" manner, however one defines "the best" and "the most precise" in terms of accuracy, it would be safe to say that no matter what definition is applied, this type of network would also be "the most expensive". Whether this cost is justified or whether a somewhat less accurate determination will be *satisfactory* is a decision the user and designer must make. If cost effectiveness is one of the externally applied factors on a design, the user and designer must also be able to quantify what "is good enough" to meet the user's requirements.

Consider two variance-covariance matrices resulting from different designs, I and II of two parameters y and z

$$\Sigma_I = \begin{bmatrix} 5 & 0 \\ 0 & 4 \end{bmatrix} ; \quad \Sigma_{II} = \begin{bmatrix} 5 & 3 \\ 3 & 4 \end{bmatrix} ; \quad X = \begin{bmatrix} y \\ z \end{bmatrix}$$

Judging from their standard deviations, if the user is interested only in X, both designs define these parameters equally well. If the user is interested in a function of parameters, t, where $t = y + z$

then the design resulting in V/C matrix Σ_I allows t to be determined with a variance of 9 while the design resulting in matrix Σ_{II} allows t to be determined with a variance of 15. Which is better, cheaper, or sufficient for the user's needs are questions the user must be able to answer.

Design of a horizontal network and required observations would be made simpler if user requirements were stated quantitatively as follows:

1. Station location accuracies with respect to a designated datum. In subsequent discussions, these will be referred to as target station variances.
2. Required estimable quantities and their largest acceptable accuracies. In the following discussion, these will be designated target estimable variances.
3. Approximate location of all points to be controlled in this network.

The above is not an all inclusive listing but even so it is probably more than can be expected from most users. The discussion that follows will assume that the a priori variances assigned to observations made with particular instruments and techniques are correct, since all the estimates of the variances of station coordinates and estimable quantities (estimables) are based upon these values. While the discussion of the estimables will be limited again to distances, azimuths, and angles within the network, the general guidelines developed will be applicable to any user specified quantity. It will also be presumed that a

preliminary reconnaissance has been performed so that intervisibility data are available.

If the user is unclear as to what estimables are required, a detailed examination of the ultimate use of the network may be the best indicator of which quantities are important in design and also which "types" of the "observations" will be required to establish the network. This may also be the key to quantifying the target variances for the required quantities. The estimables considered in this study fall into the general categories outlined in sections 2.3.2.1 through 2.3.2.3. For example, a network used to control the path of a right of way through a given area, whose control points will be used as starting and ending points for supplementary traverses will require a scale, so distance must be one of the estimables. If control points are to be subsequently recovered and, before proceeding, the surveyor on this subsequent mission is required to verify recovery by observing a "check angle" (that is, an angle between the occupied station and two other inter-visible stations in the network) estimable angles will also be required. If the location of parcels of land will also be required on the project, azimuth estimability is also necessary along with the means to transform coordinates from the pseudo-inverse implied constraint into the system of an established datum.

If only scale and angle estimability are required, as would be the case in some relative network, the "expense" of azimuth observations may be avoidable.

Once the user and designer decide upon the estimables to be established, the actual role to be played by the net control points should be examined. If, for example, check angles are to be required for subsequent work, some criterian for the predicted (computed from station location) angles within the net will be required, since the allowable deviation between the observed angle and that computed from the coordinates will set the accuracy of the predictions as compared with the accuracy of the observation.

If the network is to provide scale to purely angular subsequent observations, the final accuracy required for the subsequent work provides an ideal of the accuracy required of distances within the net. For example, if specific pairs of stations are to be used as baselines for class II, third order expansions, the standard deviations of the distances between these pairs of stations should not exceed one part in 250,000 of the distance (TABLE 2, TRIANGULATION, USDC (1974)).

If the user is only interested in estimable quantities and not in the positional accuracy of the station coordinates themselves, the station accuracies will be considered <u>nominal</u> and no additional survey effort will be expended to meet the target station variances once the criteria for the target estimable variances have been met.

<u>In subsequent discussions in this chapter, the control network will be assumed to have been established by observations and adjustment techniques which result in a V/C matrix for parameters, expressing the uncertainty in the values attributed to the parameters.</u> The effect of this uncertainty in the positional parameters will be studied as it

effects the determination of user estimables. All "observations" and instruments referred to will be those of the user in his subsequent use of the control network.

The idea developed in this chapter refers to only one of the possible bases upon which the accuracy of estimables may be based.

3.2 Criteria for estimable target variance based upon instruments.

If the estimable quantities in the network are to be used as check quantities for control extensions based upon the net (that is, observations made at the start and end of the extension which will be compared to computed values based on station coordinates to assure recovery) criteria can be developed based upon the instrumentation to be used in the subsequent extensions. It is common in practice to treat already established higher order control (in this case the network to be designed) as fixed in the determination of the values of such estimables as check angles, distances and azimuths. Since the decision as to station recovery is based not on the observation but on the difference between the observed and predicted (computed) values, the propagated variance for this quantity is:

$$d = D_{OBSERVED} - D_{computed/predicted} \tag{3.1}$$

$$\sigma^2_{OP} = \sigma^2_O + \sigma^2_P \tag{3.2}$$

where σ^2_{op} is the variance of a subsequent observation/prediction difference, d , σ^2_o is the variance of the observation, and σ^2_p is the variance of the prediction.

σ^2_p is a predicted accuracy, based on the V/C matrix of the station coordinates. For a given instrument, crew, and technique σ^2_o is assumed known a priori. In this subsection, a method will be suggested for quantifying accuracy requirements based on assumptions as to the acceptability of $D_{OBSERVED}$ as compated to $D_{computed/predicted}$.

Probably the best known rule concerning the acceptability of a measurement is the one based on rejection of any measurement deviating from some prescribed value, such as a mean, by more than three times the standard deviation of the measurement. If surveyors using the control established by the network to be designed use this rule and make the common assumption that the existing control is errorless, the size of σ^2_p can be based upon this rule and the cost of re-observation of a given measurement.

Again, the difference between measured and computed values, d , of a specified observation is used as a basis for this. If this difference is greater than $3\sigma_o$, the measurement is rejected and is to be repeated. Eq (3.2) indicates that the correct standard deviation to use is σ_{op} . Since $\sigma_{op} > \sigma_o$, that which the observer believes to be rejectable may in fact be smaller than $3\sigma_{op}$ and therefore be acceptable.

The more expensive the observation is to perform, the smaller should be the possibility that it is erroneously rejected or reperformed. If the usual assumption of normality is made for this difference, then

$$P(-3\sigma_{op} \leq d \leq 3\sigma_{op}) = 0.9974$$

The designer and user must agree upon the largest probability, acceptable to both, that a "good" measurement will be rejected. This probability, a, expressed in percent, may then be used to define the range in which d may vary and still be acceptable. Then, using the probability distribution function for the normal $n(0,1)$ distribution, the maximum magnitude acceptable in this range is A and

$$P(A) = 0.9974 - a/100$$

Then

$$\frac{d}{\sqrt{\sigma_o^2 + \sigma_p^2}}$$

is a statistic distributed as $n(0,1)$

At this upper limit

$$A = \frac{d_{max}}{\sqrt{\sigma_o^2 + \sigma_p^2}} = \frac{3\sigma_o}{\sqrt{\sigma_o^2 + \sigma_p^2}}$$

since the maximum magnitude the surveyor will allow is $3\sigma_o$.

Squaring

$$A^2 = \frac{9\sigma_o^2}{\sigma_o^2 + \sigma_p^2}$$

Then

$$\sigma_p^2 (\text{MAXIMUM}) = \frac{9 - A^2}{A^2} \sigma_o^2 = C\sigma_o^2 \qquad (3.3)$$

(Note that the lower case n has been used to denote the univariate normal distribution to avoid confusion with the symbol for the normal matrix N .)

Table 3.1 indicates the representative values for this coefficient, C . Also tabulated are the values for the 2 sigma (approximately 95% interval) level.

Table 3.1

C Values for $3\sigma_o$ and $2\sigma_o$ Rejection Criteria

α	$C(3\sigma_o)$	$C(2\sigma_o)$
5%	1.39	0.063
10%	2.38	0.505
15%	3.40	0.955
20%	4.58	1.479
25%	5.92	2.076
50%	12.43	7.908

α is the percentage probability that a blunder (greater than $3\sigma_o/2\sigma_o$) actually is less than $3\sigma_{op}/2\sigma_{op}$.

Figure 3.1 graphically illustrates two extreme situations in application of the C values from table 3.1. The graph labelled 5 percent indicates the normal probability density function indicating that the area between the $3\sigma_o/2\sigma_o$ and $3\sigma_{op}/2\sigma_{op}$ limits (that is the probability than an observation will fall between those limits) is 5 percent. The graph labelled 25 percent shown directly above it indicates the effect of an increased α value. Note that while the $3\sigma_o/2\sigma_o$ values remain at a constant distance from the central axis, the result of increasing the area between that value and the $3\sigma_{op}/2\sigma_{op}$ value is a flattening out and elongation of the probability density function graph.

As an example of the use of table 3.1, suppose that some observation between two positions has a standard deviation of 1.0. If the user decides to use the $3\sigma_o$ value as a criterian for rejection of an observation (that is $|d| > 3\sigma_o$) and will accept a 10 percent probability that an observation where $|d|$ is greater than $3\sigma_o$ is rejected when actually $3\sigma_o < |d| \leq 3\sigma_{op}$. How large may σ_p be and meet this users requirement?

If

$$\sigma_o^2 = 1.0$$

and

$$\sigma_p^2 (\text{maximum}) = C\,\sigma_o^2$$

From the $3\sigma_o$ column and the 10 percent row of table 3.1,

$$C = 2.38$$

Therefore

$$\sigma_p^2 (\text{maximum}) = 2.38\,\sigma_o^2$$
$$= 2.38$$

If the user instead of choosing the $3\sigma_o$ chooses the $2\sigma_o$ value as a rejection cutoff, for the same example, from the $2\sigma_o$ column and 10 percent row of table 3.1

$$C = 0.505$$

$$\sigma_p^2 (\text{maximum}) = 0.505$$

This σ_p^2 value is the maximum variance allowable from the prediction of the user (estimable) quantity from the values and uncertainties of the network positional parameters.

Comparison of the 5% and 25% rejection criteria

Normal Distribution curves

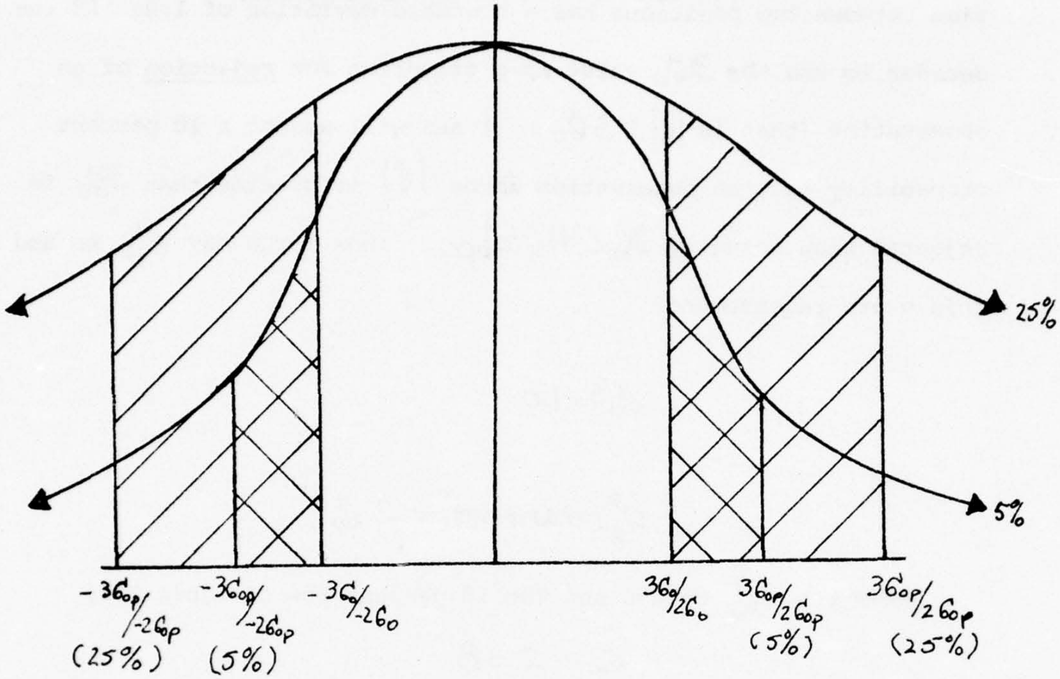


Figure 3.1

The discussion of the nature of σ_p leads to an interesting alternate interpretation of σ_{op}. As indicated in Theil (1971) in section 3.4, the user estimable may be considered as the predicted dependent value in a regression prediction. The expected value of the dependent parameter is computed from the parameters called here the positional parameters and the expectation operator. The variance of the dependent parameter as given by Theil and applied to this univariate case is nothing other than σ_{op}^2. The interpretation of table 3.1 is that the coefficient C is the ratio between the variance of the

dependent parameter when the positional parameters are considered errorless, given by Theil formula 4.8 and the variance when another unbiased estimator, in this case the predicted value from the positional parameters, is used in addition to establish the user required value. Again the α value is the percentage probability that an observation or other value of the user required quantity will fall between $\beta - 3\sigma_{op}$ and $\beta - 3\sigma_o$ or $\beta + 3\sigma_o$ and $\beta + 3\sigma_{op}$ where β may be assumed to be the expected value of the quantity in question.

3.3 Quantities other than observations.

This same approach may be taken for quantities the user is interested in which will not be directly observed. For example, suppose that traverses are to be observed using different control net points as start and end points. The user would be interested in the azimuth and/or distance between start and end points so that closure of the traverse may be checked. If the observations made in the traverse are considered independent, the expected accuracy of the azimuth and/or distance from the observations may be computed and used as the σ_o in equation 3.3, for each proposed loop. A minimum magnitude of this type of standard deviation may be defined and applied to specified pairs of stations or to all pairs of stations in the network. It should be realized that the more pairs of stations involved, however, the more "expensive" the network may be to establish.

Each quantity which can be quantified as described above or in section 3.2 will then give one user requirement. Other user requirements may be made as additions to these quantities. All these user estimable

requirements, either stated or generated as indicated will then form the basis for the definition of a V/C matrix for parameters which will be "good enough" to satisfy the user.

3.4 Choice of station variances under the $X'X$ minimum constraint.

It is entirely possible that a user requiring horizontal control for some specific use will have no station accuracy requirement, either for the $N^{\dagger}$ or some other minimum constraint. In this case the rule of thumb suggested in 2.3.2.4 cannot be used. The user estimable requirements, in terms of the variance of the estimable, offer a means to establish some nominal station variances. The "observation equations" for estimable variances given in Chapter 4, eq (2.10-12), can be used with the following simplifying assumptions:

1. All station variances, with the pseudo-inverse implied constraint, are equal.
2. All correlation coefficients in the V/C matrix are positive and equal to some "largest reasonable" value or values, for example 0.5 or 0.8.

In the next chapter, equations relating the estimable variance to those of the positional parameters will be developed. These equations, 4.4 or 4.7 depending upon the type of estimable, may be solved backwards for a sampling of the estimables and some mean or median value chosen as a first approximation for the station variances. The test for estimability outlined in section 2 can also provide the values of these variances, if used with a reasonable redundancy.

Chapter 4. A Solution for the Variance/Covariance (V/C) Matrix of Parameters Which Satisfies User Requirements

4.1 Introduction.

The computation of the variances for estimables from the variance/covariance (V/C) matrix of parameters is a clearcut problem whose solution is discussed in Chapter 2. The reverse problem, that is, the solution of a V/C matrix of parameters which will result in specified variances for estimables is less clearcut. In this chapter, a solution for this V/C matrix for parameters is outlined. This matrix will not be a function of the observations used to establish the network in which the estimables exist, but will be based solely upon the user requirements and the structure of V/C matrices in general. The ultimate use to be made of this V/C matrix will be that of a "yardstick" against which other V/C matrices, actually generated from possible observations, will be measured for adequacy.

At this point in the planning process, the following are either defined or derived characteristics concerning the net of points to be controlled:

1. The mathematical model relating positions and such observations as directions, distances and azimuths.
2. The datum in which the point coordinates are to be determined.

3. The estimables the user requires to be of a certain accuracy.

4. The target variances for these estimables and an idea of the value of the target variances for the station positions themselves.

A quote from Bossler (1972) is appropriate in this case:

> "Suppose that you are holding a potatoe in your hand and you need to know something about its weight. There is, of course, a temptation to say that you know nothing about the weight of the potato a priori. As Savage points out here, it has been impossible to give a satisfactory definition of the tempting expression "know nothing". Further, any prudent man, when obligated to mail the potato, without weighing it, will find that he knows a great deal about the weight of the potato. He would certainly provide enough postage so that it would not be returned and that it would not be overpaid by 500 percent.

As with the potato described above, we know more about the V/C matrix of parameters of the net to be established than is at first apparent. This knowledge falls into the following categories:

1. Requirements defined by the user
2. The characteristics of any V/C matrix
3. Desirable additional characteristics for the V/C matrix of parameters.

4.1.1 Requirements defined by the user.

As indicated in chapters 2 and 3, the user defines m quantities which are to be estimable. Upper limits are to be placed on the size of the variances for these estimables, either by direct requirement or consideration of the use of the network.

This, if

$$\Upsilon = F(\phi_i, \lambda_i, \phi_j, \lambda_j, \cdots, \phi_n, \lambda_n)$$

is the mathematical model relating the estimables to the station coordinates (in this case, latitude and longitude), applying the equations for the propagation of error previously given:

$$\underset{m\,m}{\Sigma_Y} \geq \frac{\partial F}{\partial(\phi,\lambda)} \Sigma_{\phi\lambda} \left[\frac{\partial F}{\partial(\phi,\lambda)}\right]'$$

$$C \equiv \frac{\partial F}{\partial(\phi,\lambda)}$$

$$\underset{m\,m}{\Sigma_Y} \geq C \Sigma_X C' \tag{4.1}$$

where the upper limit of <u>only</u> the major diagonal terms of the Σ_X matrix are known. The upper limits of the Σ_Y major diagonal terms (the user specified estimables) are also known. The off diagonal terms of the Σ_Y matrix may have any physically possible values. These off diagonal term values will be discussed further in section 4.1.2. If the off diagonal terms of the Σ_X matrix are considered unknowns, X_c, eq 4.1 represents a set of equations relating the unknowns, X_c, to the limits set for the upper bounds of the estimable variances in Σ_Y and for the off diagonal magnitudes of the Σ_Y matrix set by eq 4.2. This is easiest to visualize by considering the formation of the $C\Sigma_X C'$ product in parts. Firstly, consider the product $C\Sigma_X$ as diagrammed in Figure 4.1.

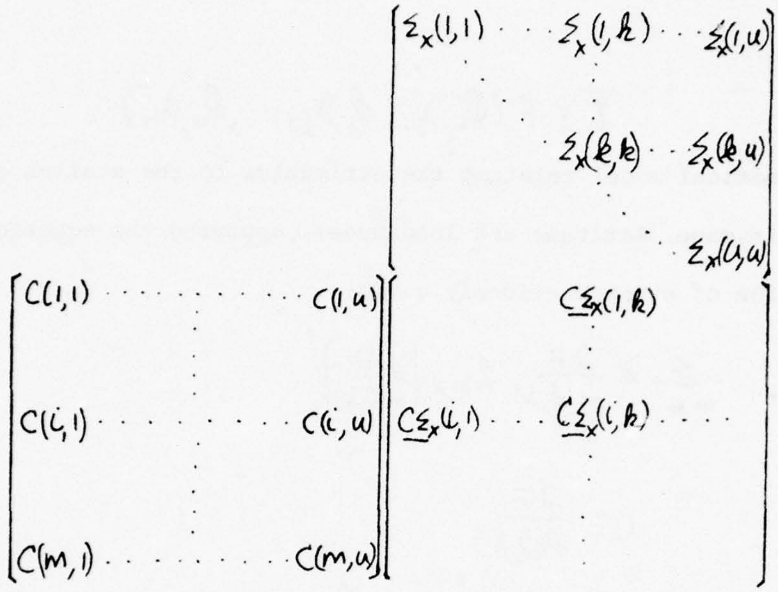


Figure 4.1

The equation for the kth element of the ith row of the $C\Sigma_x$ product is:

$$C\Sigma_x(i,k) = \sum_{h=1}^{u} C(i,h)\,\Sigma_x(h,k)$$

$$\Sigma_x(h,k) = \Sigma_x(k,h)$$

$$C\Sigma_x(i,k) = \sum_{h=1}^{k} C(i,h)\,\Sigma_x(h,k) + \sum_{h=k+1}^{u} C(i,h)\,\Sigma_x(k,h) \qquad (4.1.1)$$

which takes advantage of the symmetry of the Σ_x matrix. The second product is then $\underline{C\Sigma_x}\,C'$, diagrammed in Figure 4.2.

$$
\begin{array}{cc}
 & \begin{bmatrix} C'(1,1) & \cdots & C'(1,j) & \cdots & C'(1,m) \\ & & \vdots & & \\ C'(i,1) & \cdots & C'(i,j) & \cdots & C'(i,m) \\ & & \vdots & & \\ C'(u,1) & \cdots & C'(u,j) & \cdots & C'(u,m) \end{bmatrix} \\
\begin{bmatrix} \underline{C\Sigma_X}(1,1) & \cdots & C\Sigma_X(1,u) \\ & \vdots & \\ C\Sigma_X(i,1) & \cdots & C\Sigma_X(i,u) \\ & \vdots & \\ C\Sigma_X(m,1) & \cdots & C\Sigma_X(m,u) \end{bmatrix} & \begin{bmatrix} & & \vdots & & \\ & & \vdots & & \\ \cdots & \cdots & C\Sigma_X C'(i,j) & \cdots & \\ & & \vdots & & \\ & & \vdots & & \end{bmatrix}
\end{array}
$$

Figure 4.2

The equation for the jth element of this product, which is equal to the (i,j) element of the Σ_Y matrix is then:

$$\Sigma_Y(i,j) = \sum_{k=1}^{u} \underline{C\Sigma_X}(i,k)\, C'(k,j)$$

but $C'(k,j) = C(j,k)$

$$\Sigma_Y(i,j) = \sum_{k=1}^{u} C\Sigma_X(i,k)\, C(j,k)$$

and substituting for $\underline{C\Sigma_X}(i,k)$

$$\Sigma_Y(i,j) = \sum_{k=1}^{u} \left\{ \sum_{h=1}^{k} C(i,h)\,\Sigma_X(h,k) + \sum_{h=k+1}^{u} C(i,k)\,\Sigma_X(k,h) \right\} C(j,k) \tag{4.1.2}$$

The above equation will be simplified in application to the estimables discussed in sections 2.3.2.1 through 2.3.2.3 and will represent the relationship between the unknown off diagonal coefficients of the matrix, X_c , and the user specified major diagonal terms as well as the physically possible off diagonal of the Σ_Y matrix.

4.1.2 Required and desirable characteristics of any V/C matrix.

These characteristics refer to the magnitude of the off diagonal terms of any physically attainable V/C matrix. Simply, the correlation cofficient at any location in the V/C matrix cannot be greater than unity in magnitude, implying

$$|\Sigma(i,j)| \leq (\Sigma(i,i)\Sigma(j,j))^{1/2} \qquad (i \neq j)$$

or (4.2)

$$|\sigma_{ij}| \leq (\sigma_{ii}^2 \sigma_{jj}^2)^{1/2}$$

where σ_{ij} is the (i,j) element of Σ V/C matrix. σ_{ii}^2 and σ_{jj}^2 refer to the (i,i) and ($j\ j$) elements of that same matrix and Σ is either Σ_X or Σ_Y . All V/C matrices are symmetric, thus any equation true for σ_{ij} is true for σ_{ji} . For the V/C matrix for the parameters, if the usual meaning is taken for high correlations (that is, that they imply a poorly parametrized model), the off diagonal terms should be as small (close to zero) as practicable, while all other requirements are fulfilled.

4.2 Quantifying the requirements.

An inspection of eq. 2.11 and 2.12 indicate that they can be used to relate the estimable azimuth or distance to the position parameters and so they are not only "observation equations" but also the components of the C matrix, that is propagation equations, as well. The linearized propagation for an estimable angle can also be formed from eq. 2.10, the direction "observation equation". That is, for any angle α_{ilm}

$$\alpha_{ilm} = D_{il} - D_{im}$$

$$\delta\alpha_{ilm} = \delta D_{il} - \delta D_{im}$$

$$\begin{aligned}\delta\alpha_{ilm} = {} & \left[\frac{M_i}{S_{im}}\sin A_{im} + \frac{M_i}{S_{il}}\sin A_{il}\right]\delta\phi_i \\ & + \frac{M_l}{S_{il}}\sin A_{li}\,\delta\phi_l - \frac{M_m}{S_{im}}\sin A_{mi}\,\delta\phi_m \\ & + \left[\frac{N_m}{S_{im}}\cos\phi_m - \frac{N_l}{S_{il}}\cos\phi_l\right]\delta\lambda_i \\ & - \frac{N_m}{S_{im}}\cos\phi_m\,\delta\lambda_m + \frac{N_l}{S_{il}}\cos\phi_l\,\delta\lambda_l\end{aligned} \tag{4.3}$$

If all ${}_u\Sigma_{x_u}$ terms which are not on the major diagonal of the matrix are considered a set of unknowns, X_c, then equations 4.1.2 represent a set of $m(m+1)/2$ equations in $u(u-1)/2$ unknowns which relate the estimable variances and covariances to those of the parameters. It should be remembered that this is in fact not a set of equations but

a set of inequalities, since the user requirements will be satisfied by a $C\Sigma_x C'(i,i)$ which is smaller than $\Sigma_Y(i,i)$ and a $C\Sigma_x C'(i,j), i \neq j,$ which is physically possible. There is no unique solution, X_c , to the inequality. One of the infinite number of solutions can be used as a basis for the definition of the matrix. This solution should accomplish, in addition to satisfaction of eq 4.1 as well as possible (minimizing the corrections to the right-hand side (upper variance) bounds), the desire to have the unknowns, X_c , as close to zero as possible. This second task can be accomplished by employing a minimum norm (minimum $X'X$) solution to a set of consistent equations formed from eq 4.1. This, a usual "least squares" fit of the target variance and off diagonal magnitude terms employing the pseudo-inverse, meets the *requirements in* 4.1.

It should be reiterated at this point that although this research concentrates upon certain estimables, equation 4.1.2 is correct for any estimable quantity defined by the user. In the next section, 4.1.2 will be simplified for specific estimables.

4.2.1 The estimable/parameter relationship for specific estimables.

Equation 4.1.2 stated as an inequality as described in the previous section, is given as:

$$\Sigma_Y(i,j) \geq \sum_{k=1}^{u} \left\{ \sum_{h=1}^{k} C(i,h)\Sigma_x(h,k) + \sum_{h=k+1}^{u} C(i,h)\Sigma_x(k,h) \right\} C(j,k) \tag{4.3}$$

For the specific estimables azimuth, distance and angle, one immediate simplification is that none of these are functions of the station unknown, z, so from this point the Σ_x matrix which will be determined will contain only the variances/covariances for the latitudes and longitudes for each of the stations. A slight change of notation from matrix element form makes the physical significance of 4.1.2 more obvious. This notation change recognizes that any station i, has a latitude parameter, ϕ_i, and a longitude parameter, λ_i.

If it is assumed that the maximum number of stations involved in any estimable is three (in the case of an estimable angle) an equation of the form

$$Y_\alpha = f(\phi_i, \lambda_i, \phi_j, \lambda_j, \phi_k, \lambda_k)$$

can be directly derived from the matrix notation. Consider any two estimables α and β.

If
$$\alpha = f(\phi_i, \lambda_i, \phi_j, \lambda_j, \phi_k, \lambda_k)$$
$$\beta = f(\phi_l, \lambda_l, \phi_m, \lambda_m, \phi_n, \lambda_n)$$

$$\begin{aligned} \delta\alpha &= C_{ijk} X_c \\ \delta\beta &= C_{lmn} X_c \end{aligned} \quad ; \quad X_c = \begin{bmatrix} \Sigma_{\phi_1 \lambda_1} \\ \Sigma_{\phi_1 \phi_2} \\ \Sigma_{\phi_1 \lambda_2} \\ \vdots \\ \Sigma_{\phi_n \lambda_n} \end{bmatrix}$$

$$C_{ijk} = [0 \,\vdots\, 0 \,\vdots\, \cdots \,\vdots\, C_{\phi_i} \,\vdots\, C_{\lambda_i} \,\vdots\, \cdots \,\vdots\, C_{\phi_j} \,\vdots\, C_{\lambda_j} \,\vdots\, \cdots \,\vdots\, C_{\phi_k} \,\vdots\, C_{\lambda_k} \,\vdots\, 0 \,\vdots\, 0]$$

$$C_{lmn} = [0 \,\vdots\, 0 \,\vdots\, \cdots \,\vdots\, C_{\phi_l} \,\vdots\, C_{\lambda_l} \,\vdots\, \cdots \,\vdots\, C_{\phi_m} \,\vdots\, C_{\lambda_m} \,\vdots\, \cdots \,\vdots\, C_{\phi_n} \,\vdots\, C_{\lambda_n} \,\vdots\, \cdots \,\vdots\, 0 \,\vdots\, 0]$$

since all other coefficients than those of the i , j , k and l , m , n stations are zero.

In general, the $\Sigma_y(i,j)$ of equation 4.1.2 can be written as the $\Sigma_{\alpha\beta}$ term:

$$\Sigma_{\alpha\beta} = C_{ijk}\,\Sigma_x\,C'_{lmn}$$

$$\Sigma_{\alpha\beta} = \left(\begin{array}{l} C_{\phi_i} C_{\phi_l} \Sigma_{\phi_i\phi_l} \\ + C_{\lambda_i} C_{\phi_l} \Sigma_{\lambda_i\phi_l} \\ + C_{\phi_j} C_{\phi_l} \Sigma_{\phi_j\phi_l} \\ + C_{\lambda_j} C_{\phi_l} \Sigma_{\lambda_j\phi_l} \\ + C_{\phi_k} C_{\phi_l} \Sigma_{\phi_k\phi_l} \\ + C_{\lambda_k} C_{\phi_l} \Sigma_{\lambda_k\phi_l} \end{array}\right) +$$

$$\left(\begin{array}{l} C_{\phi_i} C_{\lambda_l} \Sigma_{\phi_i\lambda_l} \\ + C_{\lambda_i} C_{\lambda_l} \Sigma_{\lambda_i\lambda_l} \\ + C_{\phi_j} C_{\lambda_l} \Sigma_{\phi_j\lambda_l} \\ + C_{\lambda_j} C_{\lambda_l} \Sigma_{\lambda_j\lambda_l} \\ + C_{\phi_k} C_{\lambda_l} \Sigma_{\phi_k\lambda_l} \\ + C_{\lambda_k} C_{\lambda_l} \Sigma_{\lambda_k\lambda_l} \end{array}\right) + \left(\begin{array}{l} C_{\phi_i} C_{\phi_m} \Sigma_{\phi_i\phi_m} \\ C_{\lambda_i} C_{\phi_m} \Sigma_{\lambda_i\phi_m} \\ C_{\phi_j} C_{\phi_m} \Sigma_{\phi_j\phi_m} \\ C_{\lambda_j} C_{\phi_m} \Sigma_{\lambda_j\phi_m} \\ C_{\phi_k} C_{\phi_m} \Sigma_{\phi_k\phi_m} \\ C_{\lambda_k} C_{\phi_m} \Sigma_{\lambda_k\phi_m} \end{array}\right)$$

$$+ \left(\begin{array}{l} C_{\phi_i} C_{\lambda_m} \Sigma_{\phi_i\lambda_m} \\ + C_{\lambda_i} C_{\lambda_m} \Sigma_{\lambda_i\lambda_m} \\ + C_{\phi_j} C_{\lambda_m} \Sigma_{\phi_j\lambda_m} \\ + C_{\lambda_j} C_{\lambda_m} \Sigma_{\lambda_j\lambda_m} \\ + C_{\phi_k} C_{\lambda_m} \Sigma_{\phi_k\lambda_m} \\ + C_{\lambda_k} C_{\lambda_m} \Sigma_{\lambda_k\lambda_m} \end{array}\right)$$

$$
+\left\{\begin{array}{l} C_{\phi_i} C_{\phi_n} \Sigma_{\phi_i \phi_n} \\ + C_{\lambda_i} C_{\phi_n} \Sigma_{\lambda_i \phi_n} \\ + C_{\phi_j} C_{\phi_n} \Sigma_{\phi_j \phi_n} \\ + C_{\lambda_j} C_{\phi_n} \Sigma_{\lambda_j \phi_n} \\ + C_{\phi_k} C_{\phi_n} \Sigma_{\phi_k \phi_n} \\ + C_{\lambda_k} C_{\phi_n} \Sigma_{\lambda_k \phi_n} \end{array}\right\}
+\left\{\begin{array}{l} C_{\phi_i} C_{\lambda_n} \Sigma_{\phi_i \lambda_n} \\ + C_{\lambda_i} C_{\lambda_n} \Sigma_{\lambda_i \lambda_n} \\ + C_{\phi_j} C_{\lambda_n} \Sigma_{\phi_j \lambda_n} \\ + C_{\lambda_j} C_{\lambda_n} \Sigma_{\lambda_j \lambda_n} \\ + C_{\phi_k} C_{\lambda_n} \Sigma_{\phi_k \lambda_n} \\ + C_{\lambda_k} C_{\lambda_n} \Sigma_{\lambda_k \lambda_n} \end{array}\right\} \qquad (4.4)
$$

The correspondence between equation 4.4 and 4.1.2 can be seen if it is recalled that each station, i , represents two parameters ϕ_i and λ_i in equation 4.4 and two parameters $(i-1)\times 2+1$ for ϕ_i and $(i-1)\times 2+2$ for λ_i in equation 4.1.2. The u parameters in equation 4.1.2 represent the $u/2$ stations in 4.4.

If $\alpha = \beta$, that is, if the equation represents a target estimable variance, this reduces to

$$
\Sigma_{\alpha\alpha} = (C_{\phi_i})^2 \Sigma_{\phi_i \phi_i} + (C_{\lambda_i})^2 \Sigma_{\lambda_i \lambda_i} + (C_{\phi_j})^2 \Sigma_{\phi_j \phi_j} + (C_{\lambda_j})^2 \Sigma_{\lambda_j \lambda_j} + (C_{\phi_k})^2 \Sigma_{\phi_k \phi_k} + (C_{\lambda_k})^2 \Sigma_{\lambda_k \lambda_k} +
$$

$$
2\begin{pmatrix} C_{\lambda_i} C_{\phi_i} \Sigma_{\lambda_i \phi_i} \\ + C_{\lambda_k} C_{\phi_i} \Sigma_{\lambda_k \phi_i} \\ + C_{\phi_j} C_{\phi_i} \Sigma_{\phi_j \phi_i} \\ + C_{\lambda_j} C_{\phi_i} \Sigma_{\lambda_j \phi_i} \\ + C_{\phi_k} C_{\phi_i} \Sigma_{\phi_k \phi_i} \end{pmatrix} + 2\begin{pmatrix} C_{\phi_j} C_{\lambda_i} \Sigma_{\phi_j \lambda_i} \\ + C_{\lambda_j} C_{\lambda_i} \Sigma_{\lambda_j \lambda_i} \\ + C_{\phi_k} C_{\lambda_i} \Sigma_{\phi_k \lambda_i} \\ + C_{\lambda_k} C_{\lambda_i} \Sigma_{\lambda_k \lambda_i} \end{pmatrix}
$$

$$
+ 2\begin{pmatrix} C_{\lambda_j} C_{\phi_j} \Sigma_{\lambda_j \phi_j} \\ + C_{\phi_k} C_{\phi_j} \Sigma_{\phi_k \phi_j} \\ + C_{\lambda_k} C_{\phi_j} \Sigma_{\lambda_k \phi_j} \end{pmatrix} + 2\begin{pmatrix} C_{\phi_k} C_{\lambda_j} \Sigma_{\phi_k \lambda_j} \\ + C_{\lambda_k} C_{\lambda_j} \Sigma_{\lambda_k \lambda_j} \end{pmatrix}
$$

$$
+ 2 C_{\phi_k} C_{\lambda_k} \Sigma_{\phi_k \lambda_k} \tag{4.5}
$$

In the case of estimable distances combined with estimable azimuths, both of which are functions of only two stations, eq 4.1 further simplifies to:

$$
\Sigma_{\alpha\beta} = \begin{pmatrix} C_{\phi_l} C_{\phi_i} \Sigma_{\phi_l \phi_i} \\ + C_{\phi_l} C_{\lambda_i} \Sigma_{\phi_l \lambda_i} \\ + C_{\phi_l} C_{\phi_j} \Sigma_{\phi_l \phi_j} \\ + C_{\phi_l} C_{\lambda_j} \Sigma_{\phi_l \lambda_j} \end{pmatrix} + \begin{pmatrix} C_{\lambda_l} C_{\phi_i} \Sigma_{\lambda_l \phi_i} \\ + C_{\lambda_l} C_{\lambda_i} \Sigma_{\lambda_l \lambda_i} \\ + C_{\lambda_l} C_{\phi_j} \Sigma_{\lambda_l \phi_j} \\ + C_{\lambda_l} C_{\lambda_j} \Sigma_{\lambda_l \lambda_j} \end{pmatrix} +
$$

$$\left\{\begin{array}{l} C_{\varphi_k} C_{\varphi_i} \Sigma_{\varphi_k \varphi_i} \\ + C_{\varphi_k} C_{\lambda_i} \Sigma_{\varphi_k \lambda_i} \\ + C_{\varphi_k} C_{\varphi_j} \Sigma_{\varphi_k \varphi_j} \\ + C_{\varphi_k} C_{\lambda_j} \Sigma_{\varphi_k \lambda_j} \end{array}\right\} + \left\{\begin{array}{l} C_{\lambda_\ell} C_{\varphi_i} \Sigma_{\lambda_\ell \varphi_i} \\ + C_{\lambda_\ell} C_{\lambda_i} \Sigma_{\lambda_\ell \lambda_i} \\ + C_{\lambda_\ell} C_{\varphi_j} \Sigma_{\lambda_\ell \varphi_j} \\ + C_{\lambda_\ell} C_{\lambda_j} \Sigma_{\lambda_\ell \lambda_j} \end{array}\right\} \tag{4.6}$$

For an estimable target variance in this case, the above becomes:

$$\begin{aligned} \Sigma_{\alpha\alpha} = {} & (C_{\varphi_i})^2 \Sigma_{\varphi_i \varphi_i} + (C_{\lambda_i})^2 \Sigma_{\lambda_i \lambda_i} + (C_{\varphi_j})^2 \Sigma_{\varphi_j \varphi_j} \\ & + (C_{\lambda_j})^2 \Sigma_{\lambda_j \lambda_j} \\ & + 2 C_{\varphi_i} C_{\lambda_i} \Sigma_{\varphi_i \lambda_i} + 2 C_{\varphi_i} C_{\varphi_j} \Sigma_{\varphi_i \varphi_j} + 2 C_{\lambda_i} C_{\varphi_j} \Sigma_{\lambda_i \varphi_j} \\ & + 2 C_{\lambda_i} C_{\lambda_j} \Sigma_{\lambda_i \lambda_j} + 2 C_{\lambda_j} C_{\varphi_i} \Sigma_{\lambda_j \varphi_i} + 2 C_{\varphi_j} C_{\lambda_j} \Sigma_{\varphi_j \lambda_j} \end{aligned} \tag{4.7}$$

to restate the inequality for equations 4.4, 4.5, 4.6 and 4.7:

For 4.4 and 4.6:

$$|\Sigma_Y(i,j)| \geq |\Sigma_{\alpha\beta}|$$

$i \neq j$

$$|\Sigma_{\alpha\beta}| \leq \left(\Sigma_Y(i,i)\, \Sigma_Y(j,j)\right)^{1/2}$$

where i represents the α estimable and j the β estimable

For 4.5 and 4.7:

$$\Sigma_{\alpha\alpha} \leq \Sigma_Y(i,i)$$

the target for the ith estimable, α .

4.3 Formation of the equations defining the unknown coefficients,

Equations 4.4 through 4.7 can be represented as the design matrix for the unknown off diagonal coefficients, A_c . Since they are already "linear" once the C matrix coefficients have been evaluated at the approximate values for the coordinates of the station, this $m(m+1)/2$ by $u(u-1)/2$ matrix representing them will be a mix of the linearized forms of these equations representing estimables of different types. If angles are one of the estimables, equation 4.4 must be used to evaluate the off diagonal terms of the Σ_Y matrix. In the case of an off diagonal term representing the covariance between an angle and either a distance or azimuth, the coefficients of the station n will be zero. The coefficients of the stations ℓ and m will be those from the "observation equation" describing the estimable which involves stations ℓ and m . A set of $u(u-1)/2$ equations which are consistent can then be formed as:

$$A_c' P_c A_c \equiv N_c \ ; \quad A_c' P_c W_c \equiv U_c$$

$$N_c X_c = -U_c$$

where P_c is a weight ascribed to each of the $m(m+1)/2$ variance/covariance s.

4.4 The unknown off diagonal terms.

Equation (4.4) and (4.6) relate target positional parameter variances $\underset{u\ u}{\Sigma_X}(i,i)$ and off diagonal coefficients, X_C in the Σ_x matrix to variances and off diagonal terms in the $\underset{m\ m}{\Sigma_Y}$ matrix. While an $X_C' X_C$ minimum solution could be performed which would produce corrections to both the station target variances and off diagonal terms, control of the magnitude of the target station variances would be at best very difficult. These major diagonal terms, more than any others in the Σ_X matrix, must be close to the physically attainable situation. This would be possible by considering the target positional parameter variances as weighted heavily at their a priori values, but the difference between heavy weighting and absolute constraint is minor. Therefore, these target variances will be constrained at their a priori values and removed from the "unknown" category. Since the $\underset{u\ u}{\Sigma_X}$ matrix is a V/C matrix, the remaining off diagonal terms are symmetric, and this symmetry condition is also to be enforced absolutely. There are $u(u-1)/2$ remaining coefficients which are treated as unknowns. If the approximate values, X_{C^o}, for these unknowns are taken as zero, the correction vector from a minimum-correction-to-target-variance solution which minimizes the sum of the squares of the corrections to the X_C is the vector of the parameter (unknown off diagonal term) values. The numbering scheme chosen for these X_C parameters is indicated in the diagram below:

$$\underset{u\,u}{\Sigma_X} = \begin{array}{c} \begin{array}{cccccccc} \phi_1 & \lambda_1 & \phi_2 & \lambda_2 & \cdots & \cdots & \phi_h & \lambda_h \end{array} \\ \left[\begin{array}{cccccccc} (1) & (2) & (3) & \cdots & \cdots & \cdots & (u-2) & (u-1) \\ & (u) & (u+1) & \cdots & \cdots & \cdots & \cdots & (2u-3) \\ & & (2u-1) & \cdots & \cdots & \cdots & \cdots & (3u-6) \\ & & & \ddots & & & & \vdots \\ & & & & \ddots & & & \vdots \\ & & & & & \ddots & & \vdots \\ & & & & & & \ddots & \vdots \\ & & & & & & & L \end{array} \right] \end{array} \begin{array}{c} \\ \phi_1 \\ \lambda_1 \\ \phi_2 \\ \lambda_2 \\ \vdots \\ \vdots \\ \phi_h \\ \lambda_h \end{array} \qquad h \equiv \frac{u}{2}$$

where the number in parenthesis (i) corresponds to the unknown X_c (i). The total number of unknowns, L , in the case of h stations is:

$$L = h(2h-1) = u(u-1)/2$$

(Note that there are two parameters to be considered for each station in this evaluation).

As can be seen from this equation, the number of unknowns is very large for even small networks. This number can be decreased if the estimables are examined in detail. Within the net, if no estimable shares the i and j stations, the minimum $X_c' X_c$ contribution for the correlation between these stations is at the zero value for the coefficients which relate stations i and j . These terms will also be removed from the parameter vector and constrained at an a priori value of zero. This is called "decoupling" in subsequent discussions.

4.5 Evaluation of misclosures, W_c .

The misclosure, which corresponds to the vector W_c in the equations:

$$A_c X_c^o + W_c = 0$$

and

$$u_c = A_c' P_c W_c$$

is evaluated in an unusual manner because the mathematical model for this linearized equation is an inequality. The case where this corresponds to an estimable angle which is a function of stations i , j , and k is treated as the general case. The evaluation of an estimable distance or azimuth is performed in exactly the same manner except that the contribution to W_c from the third station, k , is zero.

For a major diagonal element, the a priori values for the unknowns, X_c^o , are zero, the major diagonal terms, modelled in equation set (4.5), reduce to:

$$\Sigma_{\alpha\alpha}\Big|_{X_c^o} = (C_{\phi_i})^2 \Sigma_{\phi_i\phi_i} + (C_{\lambda_i})^2 \Sigma_{\lambda_i\lambda_i} + (C_{\phi_j})^2 \Sigma_{\phi_j\phi_j} + (C_{\lambda_j})^2 \Sigma_{\lambda_j\lambda_j} + (C_{\phi_k})^2 \Sigma_{\phi_k\phi_k} + (C_{\lambda_k})^2 \Sigma_{\lambda_k\lambda_k}$$

Since the $\Sigma_{\phi\phi}$ and $\Sigma_{\lambda\lambda}$ elements are treated as known, the inequality becomes:

$$\Sigma_{\alpha\alpha}\Big|_{X_c^o} \leq \text{ defined } \Sigma_y(i,i) \text{ variance value, } \Sigma_{\alpha\alpha}$$

where α is the i^{th} estimable.

If this is correct upon evaluation, the misclosure contribution W_c of the vector $\mathcal{U}_c$ is zero (that is, the approximate values fulfill the required conditions). If not, $W_c = \Sigma_Y(i,i) - \Sigma_{\alpha\alpha}\big|_{X_c^\circ}$

Similarly, for an off diagonal term in the Σ_Y matrix,

$$|\Sigma_{\alpha\beta}|\Big|_{X_c^\circ} \leq \left(\Sigma_Y(i,i)\,\Sigma_Y(j,j)\right)^{1/2}$$

where the j'th estimable is β .

Only if the estimable quantities involved in rows α and β share a station in common will the $|\Sigma_{\alpha\beta}| \neq 0$.

In these non-zero cases if

$$|\Sigma_{\alpha\beta}|\Big|_{X_c^\circ} \leq \left(\Sigma_Y(i,i)\,\Sigma_Y(j,j)\right)^{1/2}$$

the W_c element will be zero. If not, $W_c = \left(\Sigma_Y(i,i)\,\Sigma_Y(j,j)\right)^{1/2} - \Sigma_{\alpha\beta}\big|_{X_c^\circ}$

4.6 The assignment of weights, P_c .

From examination of the magnitudes of the coefficients in each equation from sets (4.4) through (4.7) it is obvious that many more equations express the condition of eq (4.2) than do the target estimable variances. A weight upon the former is required if the magnitude of the correction to the estimable target variance is to be controlled and held small. In addition, the W_c contribution to $\mathcal{U}_c$ from the estimable distance misclosure (in meters squared) is considerably smaller in magnitude than the contribution of an estimable angle or azimuth. In order to keep the corrections to these two quantities small, a weight differential may be required in the form of a P_c matrix, $m \times m$ (m is the number of user specified estimables).

To enforce the magnitude condition

$$\left|\Sigma_x(i,j)\right| \leq \left(\Sigma_x(i,i)\, \Sigma_x(j,j)\right)^{1/2}$$

weights on the associated a priori estimates of the unknowns can also be used, P_{cx}.

4.7 Computation of the off diagonal terms of the Σ_x matrix.

The solution of the set of equations generated by considering user requirements, as previously mentioned, minimizes the squared (weighted) sum of the corrections to the off diagonal terms and target variance of estimables conditions and then, sequentially minimizes the (weighted) sum of the squares of the coefficients of the Σ_x matrix, X_c.

Thus:

$$\left[(A_c' P_c A_c) + P_{cx}\right] X_c + U_c = 0$$

$$N_{cx} \equiv A_c' P_c A_c + P_{cx}$$

implies

$$\hat{X}_c = - N_{cx}^{\dagger} U_c$$

The N_{cx} matrix is singular (positive semidefinite) and is not the type of matrix characterized by Pope as property A (Pope (1971)). The degree of singularity is also not clear (geometrically). The most simple means of generating $N_{cx}^{\dagger}$ is by applying theorem 1.88, Graybill (1969). That is, there exist matrices P and D such that

$$D = P' N_{cx} P$$

where D is a diagonal matrix whose diagonal elements are either positive or zero (also theorem 12.2.1 since all major diagonal elements are greater than zero and N_{cx} is singular).

P is an orthogonal matrix, thus

$$\underline{P}\,\underline{P}' = I \;, \quad N_{cx} = P D \underline{P}'$$

Note that D is assumed to be arranged in decreasing size of the major diagonal elements from the upper left corner. If the rank of the N_{cx} matrix is $L_r < L$, (see section 4.4), then

$$\underset{L\,L}{D} = \begin{bmatrix} \underset{L_r L_r}{D} & \bar{0} \\ \bar{0} & \bar{0} \end{bmatrix} = \begin{bmatrix} D_{11} & & & & & & & \\ & D_{22} & & & & \bar{0} & & \\ & & D_{33} & & & & & \\ & & & D_{44} & & & & \\ & & & & \ddots & & & \\ & \bar{0} & & & & D_{L_r L_r} & & \\ & & & & & & 0 & \\ & & & & & & & \ddots \; 0 \end{bmatrix}$$

Also

$$D^{+} = \begin{bmatrix} 1/D_{11} & & & & & \\ & 1/D_{22} & & \bar{0} & & \\ & & 1/D_{33} & & & \\ & & & \ddots \; 1/D_{L_r L_r} & & \\ & \bar{0} & & & 0 & \\ & & & & & \ddots \; 0 \end{bmatrix}$$

(theorem 6.2.15, Gray bill (1969))

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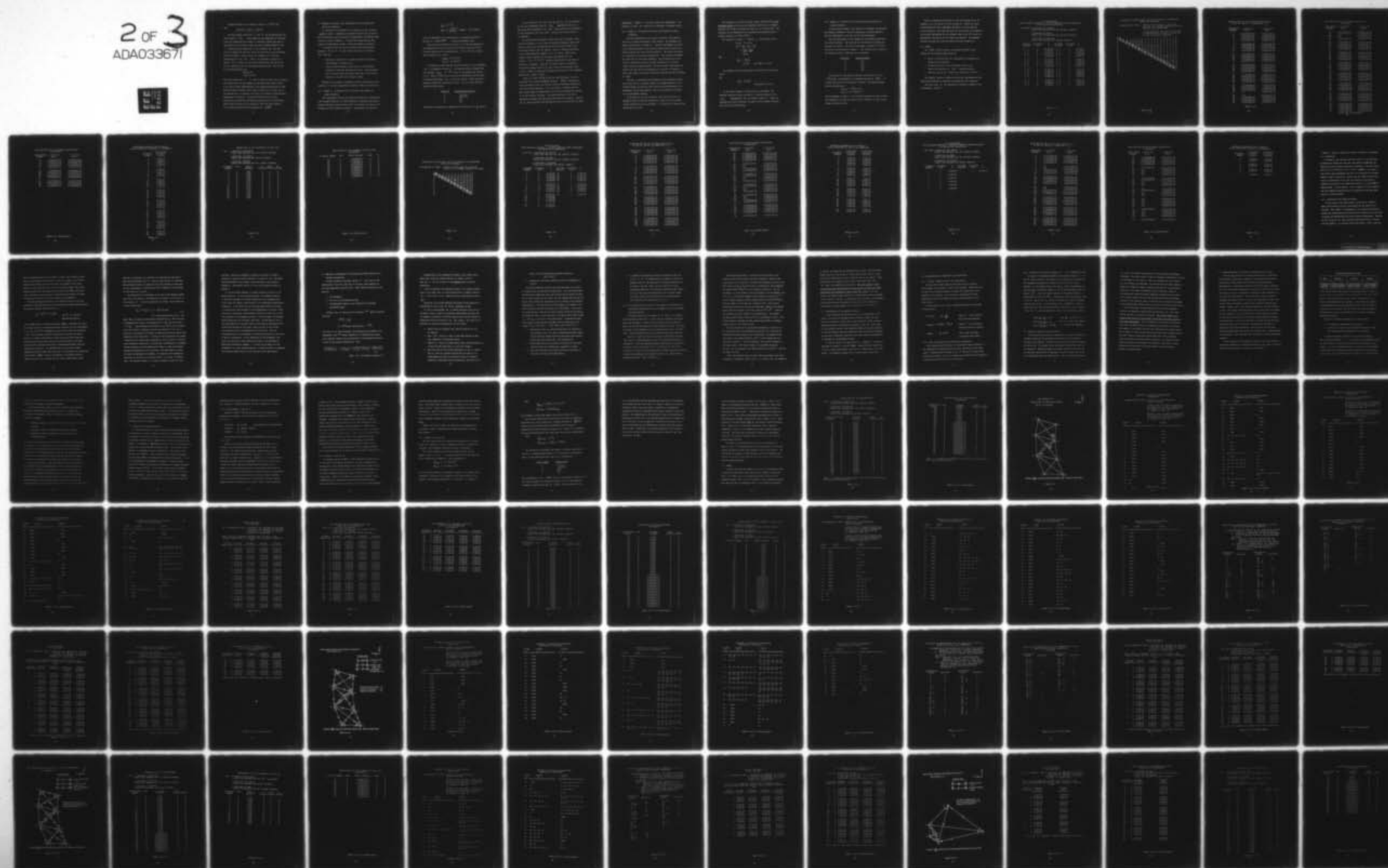


Applying theorem 6.2.10 (Graybill (1969)), if $P D P' = N_{cx}$ then

$$(P D P')^{+} = (N_{cx})^{+} = P D^{+} P'$$

The most obvious choices for P and D are the Eigenvectors and Eigen values of N_{cx}. Most schemes for the computation of the eigenvalues and eigenvectors are subject to numerical roundoff and loss of significance, so the choice of when the major diagonal elements of the D matrix actually become zero is not a clearcut one. The best system for making this decision, after the guidelines for the particular method and number of digits carried are examined, is to test the characteristics of the N_{cx}^{+} matrix. As indicated in chapter 2, a minimum $X_c' X_c$ solution for the normal equations must satisfy two of the four characteristics of the pseudo-inverse solution. The two used in the referenced proof are

$$N^{+} N N^{+} = N^{+}$$

$$(N N^{+})' = N N^{+}$$

The matrix formed from P, D may be tested to assure that it complies with either those two (many members of the generalized inverse family) or all four of those characteristics (the unique pseudo-inverse) to some specified number of digits (this number of digits is, of course, up to the designer but, at a minimum, four digits for ordinary single precision computation is suggested). The matrix formed from the a priori variances for the station latitudes and longitudes and the coefficients found in this solution will be referred to from this point onward as the "criterian V/C matrix for the parameters", $\Sigma_x(\mathrm{CRIT})$.

4.8 Examples for specific user requirements of the criterian V/C matrix of parameters.

To illustrate the techniques for the formation of the criterian $\Sigma_x(CRIT)$ matrix, 3 examples are presented for test nets T3 and T4. The configuration, station coordinates (approximate) and possible observations are given in Chapter 1. The nets were chosen because they represent configurations similar to those encountered in practice.

It should be noted here that the variances for the positional parameters ϕ and λ for each station are given in two forms. These forms are:

1. Variances in seconds of arc squared referring to latitude and longitude in seconds of arc.
2. Units of length referred to the surface of the reference ellipsoid at some mean latitude for the net. This particular unit is chosen because the author feels that it will be more familiar to the user of the control network.

Therefore, the problem is formulated in terms of the type of units outlined in 2, but the computations are made in terms of the units of 1.

4.8.1 Example 1: T3 network with all distances and azimuths as target estimables.

User requirements are that the station accuracies be no poorer than 15cm (standard deviation) in either latitude or longitude, based upon a minimum constraint solution fixing one of the stations, for points in T3. Estimables are the azimuth and distance. Requirements for these are:

$$\Sigma_{Az} \le 0''^{2}.4$$

$$\Sigma_{DIST} \le \left[\frac{S^{2/3}}{5\times10^{5}}\right]^{2} \text{ meters}^{2} \quad (S \text{ in meters})$$

along any observable line (corresponding to a standard deviation of one part in $50{,}000\,\sqrt[3]{S^{km}}$ along any observable line).

Using the rule of thumb from section 2.3.2.4 for the magnitude of the pseudo-inverse station variances, the station variances in this criterian matrix are set to approximately (5 cm)^2 at this latitude (mean latitude of T3). Thus:

$$\Sigma_{x\,\phi_i\phi_i} \simeq 0''^{2}.16 \times 10^{-5}$$
$$\Sigma_{x\,\lambda_i\lambda_i} \simeq 0''.28 \times 10^{-5}$$

An evaluation of the computed variance misclosure of the estimables, W_c , is presented in Table 4.1. The maximum variance computed from this diagonal $\Sigma_x\big|_{X_c^0}$ is $1''^{2}.9$ of arc on the azimuth from station 5 to 6. Table 4.2 indicates the location of parameters from the vector of unknown coefficients, X_c , in the Σ_x matrix. A zero indicates a decoupled coefficient constrained to zero. A list of those decoupled stations is given below:

Station #	Decoupled Form Station #
1	5,6,7,8,9
2	5,6,7,8,9
3	8,9
4	8,9
5	8,9

(Note that the decoupling is symmetrically reflected in the Σ_x matrix).

In the formation of the N_{CX} and U_C matrices, all the distances met the user requirement with the $\Sigma_X|_{X_c^o}$ (approximate) matrix, so that while they contributed to the N_{CX} matrix they did not contribute to the misclosure or the U_c vector. This was also true of three of the azimuths.

Two solutions were performed, called the A and B solutions. The A solution is based on a fit of only the estimable variances, while the B solution contains the contributions from the criterian forced on the off diagonal terms of the Σ_Y matrix. Table 4.3 indicates the effect on the largest magnitude elements of the X_c matrix. While these elements stayed relatively of the same order of magnitude, the $|\Sigma_Y(i,j)| \leq (\Sigma_Y(i,i)\,\Sigma_Y(j,j))^{1/2}$ condition added further large terms to the vector of unknowns. Large here is used to refer to correlation coefficients computed for the Σ_X matrix which have magnitudes close to unity. Table 4.4 indicates the predicted variances for the estimables based on the $\Sigma_X(CRIT)$ matrix.

These Σ_Y values conform to the user specifications, as do the set values of the station parameter variances. $\Sigma_X(CRIT)$ represents an initial estimate of what the final V/C matrix of parameters should look like after design completion. Up to this point, no mention has been made of what types of observations will be used to establish the network, or of their quality (accuracy). This particular matrix may not be physically attainable with the assets allocated for the project. However, any Σ_X matrix obtained from observations within the allocation and which

approximates $\Sigma_x(R|T)$ will also satisfy user requirements. The formation of such a Σ_x matrix will be discussed in Chapters 5 and 6.

4.8.2 Example 2: T4 Network with angles and distances as target estimables.

The user requirements for the points in T4 and a 10cm standard deviation in station coordinates (inner origin). The estimables are angles and distances, see Table 4.5. The user requirements are based on all observable angles at each station formed from any direction to another station and a given initial direction. The initial directions are indicated in the section describing T4. The distance requirements are based upon all observable distances. The T4 stations are to be used as a foundation for lower accuracy positioning in a relative (inner) origin and azimuth system. The user accuracy requirements are based on the fact that the estimables are to be used in checking to assure that proper net station recovery was made and that the T4 network is stable.

The user, in considering the economics of this subsequent survey, has decided that if a surveyor measures an estimable quantity in net T4 which differs by more than three times the standard deviation of a measurement (set of measurements), this will be considered a blunder and the measurement repeated.

If an angle set is to be discarded, there should be only a 5% probability that it would be acceptable if errors in the T4 network station positions were also considered. If a distance is rejected, this percentage will be 10%.

The instrument to be used to measure angles and directions in the subsequent survey will be one which measures directions to a standard deviation of 1".6 and 8 positions will make up a set of directions. The distances will be measured with an instrument and procedure giving a standard deviation of $0.017 + 5^{m}/10^{6}$.

For an angle at station i between j (the initial) and k

$$\alpha_{ijk} = D_{ik} - D_{ij}$$

$$\Sigma_{\alpha\alpha} = \sigma^2_{D_{ik}} + \sigma^2_{D_{ij}} \equiv \sigma^2_{\alpha}$$

$$= \left(\frac{1.6}{\sqrt{n}}\right)^2 + \left(\frac{1.6}{\sqrt{n}}\right)^2$$

$$= 0''^2.64$$

Then

$$\sigma^2_{p\alpha} = 1.39\,\sigma^2_{\alpha}$$

$$= 0''^2.89 \qquad \text{from Table 3.1 at 5\%}$$

The distances have variances which are functions of the distance itself.

Then

$$\sigma^2_{ps} = 2.33\,\sigma^2_{s}$$

from Table 3.1 at 10%

In this small network, no stations will be decoupled. The parameter location is given in Table 4.6. The misclosures for the $\Sigma_x|_{x_0}$ (approximate), W_c are listed in Table 4.7. Table 4.8 indicates the A and B solutions, and Table 4.9/10 estimable variances based upon the $\Sigma_x(crit)$ matrix.

4.8.3 Example 3: T4 network with selected angles and distances as target estimables.

In the previous examples, large numbers of estimables are specified. The example is designed to show the application to smaller number of specific requirements of the Σ_X matrix definition method.

The user requirements in this example are for specific angles and one specific distance. The variances for the station in latitude and longitude are nominal. The same 5% requirement is placed on the angle variances as was discussed in Example 2. The following are a listing of the angles upon which a limitation is to be placed:

At Station	Between Stations
1	2-5
2	1-3
3	2-4
4	2-3
5	1-2
6	1-2

The variance for the predicted distance from station 1 to 5 is $0.^{m}00168$ (corresponding to a standard deviation of $0.^{m}041$ or one part per million of the distance in meters). The nominal station location variances are:

$$\Sigma_{X\,\varphi_i\varphi_i} \sim 0.''^{2}65 \times 10^{-6}$$
$$\Sigma_{X\,\lambda_i\lambda_i} \sim 0.''^{2}108 \times 10^{-5}$$

which correspond to roughly a 2.5cm standard deviation for both latitude and longitude at the mean latitude of the T4 network in an inner origin-azimuth coordinate system.

Again, no station was decoupled, so that the location of the X_c unknowns in the Σ_X matrix is given by Table 4.6. Table 4.11 shows the misclosures for this requirement and Table 4.12 gives both the A and B solutions. Note that the effect on the solution of the addition of equations representing the off diagonal terms to the Σ_Y matrix is approximately the same as in the example one solution. The estimable variances for both the A and B solution are given in Table 4.13.

4.9 Summary

The $\Sigma_X(CRIT)$ matrix formed in the manner described in this chapter has the following characteristics:

1. Meets or is better than user requirements on estimables and target station variances.
2. Minimizes the size of the off diagonal terms in the matrix, by enforcing the $X_c' X_c$ minimum condition.
3. Both the Σ_X and Σ_Y matrices are structurally possible.

The $\Sigma_X(CRIT)$ matrix is formed with only user requirements and the observations required to establish the network were not considered.

From this point, any Σ_X developed as indicated in Chapter 4 will be designated $\Sigma_X(CRIT)$.

THE MISCLOSURE
(USER REQUIRED VARIANCE - VARIANCE COMPUTED FROM APPROXIMATE
V/C MATRIX FOR PARAMETERS)

TYPE KEY- A INDICATES AN AZIMUTH
(UNITS FOR VARIANCE ARE ARC SECONDS SQUARED)

G INDICATES AN ANGLE
(UNITS FOR VARIANCE ARE ARC SECONDS SQUARED)

S INDICATES A DISTANCE
(UNITS FOR VARIANCE ARE METERS SQUARED)

STATIONS INVOLVED		ESTIMABLE TYPE	W C	STATIONS INVOLVED		ESTIMABLE TYPE	W C
2	1	A	-6.43E-02	2	1	S	0.00E+00
3	1	A	-5.16E-02	3	1	S	0.00E+00
4	1	A	-4.04E-02	4	1	S	0.00E+00
3	2	A	-2.63E-01	3	2	S	0.00E+00
4	2	A	0.00E+00	4	2	S	0.00E+00
4	3	A	-5.17E-03	4	3	S	0.00E+00
5	3	A	-1.09E-01	5	3	S	0.00E+00
6	3	A	-8.38E-02	6	3	S	0.00E+00
5	4	A	-4.46E-01	5	4	S	0.00E+00
6	4	A	0.00E+00	6	4	S	0.00E+00
7	4	A	-4.58E-02	7	4	S	0.00E+00
6	5	A	-1.59E+00	6	5	S	0.00E+00
7	5	A	-5.19E-01	7	5	S	0.00E+00
7	6	A	-5.00E-02	7	6	S	0.00E+00
8	6	A	0.00E+00	8	6	S	0.00E+00
9	6	A	-2.82E-01	9	6	S	0.00E+00
8	7	A	-3.10E-01	8	7	S	0.00E+00
9	7	A	0.00E+00	9	7	S	0.00E+00
9	8	A	-1.21E-01	9	8	S	0.00E+00

Table 4.1

LOCATION OF COEFFICIENTS IN THE V/C MATRIX OF PARAMETERS WHICH ARE UNKNOWNS

EXPLANATION OF TABLE - SYMBOLS AT THE START OF EACH ROW INDICATE THE UNKNOWN PARAMETER TO WHICH THAT ROW AND COLUMN REFER

A ZERO INDICATES THAT THIS COEFFICIENT IS CONSTRAINED AT A VALUE OF ZERO AND IS NOT AN UNKNOWN (DECOUPLED)

$$
\left[\begin{array}{cccccccccccccccccc}
\phi_1 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & \lambda_1 & 8 & 9 & 10 & 11 & 12 & 13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & \phi_2 & 14 & 15 & 16 & 17 & 18 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & & \lambda_2 & 19 & 20 & 21 & 22 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & & & \phi_3 & 23 & 24 & 25 & 26 & 27 & 28 & 29 & 30 & 31 & 0 & 0 & 0 & 0 \\
 & & & & & \lambda_3 & 32 & 33 & 34 & 35 & 36 & 37 & 38 & 39 & 0 & 0 & 0 & 0 \\
 & & & & & & \phi_4 & 40 & 42 & 43 & 44 & 45 & 46 & 0 & 0 & 0 & 0 & 0 \\
 & & & & & & & \lambda_4 & 47 & 48 & 49 & 50 & 51 & 52 & 0 & 0 & 0 & 0 \\
 & & & & & & & & \phi_5 & 53 & 54 & 55 & 56 & 57 & 0 & 0 & 0 & 0 \\
 & & & & & & & & & \lambda_5 & 58 & 59 & 60 & 61 & 0 & 0 & 0 & 0 \\
 & & & & & & & & & & \phi_6 & 62 & 63 & 64 & 65 & 66 & 67 & 68 \\
 & & & & & & & & & & & \lambda_6 & 69 & 70 & 71 & 72 & 73 & 74 \\
 & & & & & & & & & & & & \phi_7 & 75 & 76 & 77 & 78 & 79 \\
 & & & & & & & & & & & & & \lambda_7 & 80 & 81 & 82 & 83 \\
 & & & & & & & & & & & & & & \phi_8 & 84 & 85 & 86 \\
 & & & & & & & & & & & & & & & \lambda_8 & 87 & 88 \\
 & & & & & & & & & & & & & & & & \phi_9 & 89 \\
 & & & & & & & & & & & & & & & & & \lambda_9
\end{array}\right]
$$

Table 4.2

SOLUTION FOR THE OFF-DIAGONAL COEFFICIENTS IN THE V/C MATRIX OF PARAMETERS (X_C)

COEFFICIENT NUMBER	A SOLUTION VALUE	B SOLUTION VALUE
1	-5.1043315E-08	-2.0138214E-07
2	2.1309768E-07	1.6119259E-07
3	-7.4917281E-09	1.6285921E-06
4	2.3425912E-07	-1.4588835E-07
5	1.9158620E-07	1.8832293E-07
6	6.2083814E-08	-4.3414468E-07
7	-1.3242931E-07	6.1206009E-07
8	-7.8523250E-09	-2.1119149E-07
9	2.7596103E-10	-4.5471631E-07
10	1.9131244E-07	3.4267760E-08
11	1.5647493E-07	-1.2585667E-07
12	-1.3239605E-07	3.6219120E-08
13	2.8241425E-07	6.7085978E-08
14	2.9598408E-07	1.5130345E-06
15	1.5043008E-07	-1.1355394E-07
16	-3.0117479E-07	2.5737296E-07
17	-2.1032580E-08	-4.2308278E-07
18	1.2154167E-08	6.0190308E-07
19	-3.0110346E-07	7.3629616E-07
20	6.0280780E-07	5.8101932E-07
21	1.2105890E-08	5.3006568E-07
22	-7.0004624E-09	-2.6928291E-07
23	3.6232552E-07	-5.2254954E-08
24	-3.9226955E-10	-1.2958486E-07
25	6.3544212E-11	6.0695032E-07
26	1.7224124E-07	4.7016255E-07
27	-1.8837756E-07	1.7445135E-08
28	9.2437240E-09	3.1018027E-07
29	-6.4827589E-08	1.2696950E-07
30	0.0	3.2936953E-07
31	0.0	-2.1250071E-07
32	6.2916186E-11	-4.5754479E-08
33	-1.2382186E-11	2.7511817E-07
34	-1.8821987E-07	-1.6439481E-07
35	2.0584793E-07	2.5218935E-07

Table 4.3

SOLUTION FOR THE OFF-DIAGONAL COEFFICIENTS
(CONTINUED)

COEFFICIENT NUMBER	A SOLUTION VALUE	B SOLUTION VALUE
36	-6.4827418E-08	-4.7679613E-08
37	4.5456744E-07	-8.2716895E-08
38	0.0	-1.4243051E-07
39	0.0	2.1941895E-07
40	-2.2862923E-07	8.5199127E-07
41	5.3086438E-07	5.8336713E-07
42	4.5712494E-07	4.6119823E-07
43	-1.5096077E-07	4.4833052E-07
44	-9.2368055E-08	3.1023853E-07
45	7.9517148E-10	2.5095483E-07
46	-1.5065652E-08	-8.6730495E-08
47	4.5668668E-07	5.0127170E-07
48	3.9325863E-07	6.7213000E-07
49	-9.2130051E-08	3.8858457E-07
50	-5.6367433E-08	4.1718090E-07
51	-1.5065993E-08	1.6441794E-07
52	2.8544611E-07	-1.5868284E-07
53	-1.5367380E-07	-8.0220104E-07
54	1.1302182E-06	7.3945523E-07
55	3.7933688E-07	4.3161447E-07
56	7.3573636E-07	5.6736110E-07
57	-4.9480951E-07	-3.9441483E-07
58	3.7848577E-07	3.9188262E-07
59	1.2703049E-07	4.4854710E-07
60	-4.9414535E-07	-4.0012355E-07
61	3.3233056E-07	4.8874398E-07
62	-2.3724459E-07	3.7113978E-07
63	1.7186937E-07	-8.3248381E-08
64	-3.7026567E-08	-5.6155602E-07
65	8.5926786E-08	-5.8615706E-07
66	-8.4634337E-08	-1.1943047E-07
67	1.7007345E-08	-5.5167618E-07
68	1.3653039E-07	-3.0874941E-07
69	-3.6740502E-08	2.9339918E-07
70	7.9140747E-09	-9.4290044E-08
71	-8.4511953E-08	-9.0790081E-08
72	8.3229565E-08	-3.0615820E-07
73	1.3653153E-07	-1.6374589E-08
74	1.0961803E-06	4.7111644E-07
75	2.5423236E-07	-2.4701563E-07

Table 4.3 (Continued)

SOLUTION FOR THE OFF-DIAGONAL COEFFICIENTS
(CONTINUED)

COEFFICIENT NUMBER	A SOLUTION VALUE	B SOLUTION VALUE
76	6.3892969E-08	-1.4492213E-07
77	2.7274461E-07	4.8609462E-07
78	5.3266763E-08	-2.7983646E-07
79	2.1148519E-08	-5.5540954E-07
80	2.7273421E-07	1.4151971E-07
81	1.1642951E-06	5.3604282E-07
82	2.1052589E-08	1.8753198E-08
83	8.3530409E-09	-8.1957654E-07
84	-1.0297663E-07	5.0685986E-07
85	4.0139133E-07	2.7605637E-07
86	-8.4750980E-08	-1.4187754E-06
87	-8.5374950E-08	5.9671407E-07
88	1.8026345E-08	-7.6039464E-07
89	-7.3259855E-08	-1.2318842E-06

Table 4.3 (Continued)

PREDICTED VARIANCES FOR ESTIMABLES
USING CRITERIAN V/C MATRIX OF PARAMETERS

ESTIMABLE NUMBER	B SOLUTION ESTIMABLE VARIANCE
1	4.00E-01
2	4.00E-01
3	4.00E-01
4	4.00E-01
5	1.63E-01
6	4.00E-01
7	4.00E-01
8	4.00E-01
9	4.00E-01
10	3.23E-01
11	4.00E-01
12	4.00E-01
13	4.00E-01
14	4.00E-01
15	2.52E-01
16	4.00E-01
17	4.00E-01
18	3.28E-01
19	4.00E-01
20	3.15E-03
21	2.82E-03
22	2.91E-03
23	3.33E-03
24	3.15E-03
25	3.16E-03
26	3.17E-03
27	3.06E-03
28	3.25E-03
29	3.44E-03
30	3.05E-03
31	3.25E-03
32	3.05E-03
33	3.11E-03
34	2.81E-03
35	3.08E-03
36	2.99E-03
37	2.98E-03
38	3.02E-03

Table 4.4

DESCRIPTION OF THE ESTIMABLES IN THIS TEST

KEY - A INDICATES AN AZIMUTH
(UNITS FOR VARIANCE ARE ARC SECONDS SQUARED)

S INDICATES A DISTANCE
(UNITS FOR VARIANCE ARE METERS SQUARED)

G INDICATES AN ANGLE
(UNITS FOR VARIANCE ARE ARC SECONDS SQUARED)

ESTIMABLE NUMBER	TYPE	TARGET VARIANCE	AT STATION	FROM STATION	TO STATION
1	G	0.89	1	2	5
2	G	0.89	1	2	6
3	G	0.89	2	1	3
4	G	0.89	2	1	4
5	G	0.89	2	1	5
6	G	0.89	2	1	6
7	G	0.89	3	2	4
8	G	0.89	3	2	6
9	G	0.89	4	2	3
10	G	0.89	4	2	6
11	G	0.89	5	1	2
12	G	0.89	5	1	6
13	G	0.89	6	1	2
14	G	0.89	6	1	3
15	G	0.89	6	1	4
16	G	0.89	6	1	5

Table 4.5

DESCRIPTION OF THE ESTIMABLES IN THIS TEST
(CONTINUED)

ESTIMABLE NUMBER	TYPE	TARGET VARIANCE	FROM	TO
17	S	0.88X10**-2	1	2
18	S	0.79X10**-2	1	5
19	S	0.12X10**-1	1	6
20	S	0.17X10**-2	2	3
21	S	0.14X10**-2	2	4
22	S	0.64X10**-2	2	5
23	S	0.30X10**-2	2	6
24	S	0.13X10**-2	3	4
25	S	0.19X10**-2	3	6
26	S	0.18X10**-2	4	6
27	S	0.42X10**-2	5	6

Table 4.5 (Continued)

LOCATION OF COEFFICIENTS IN THE V/C MATRIX OF PARAMETERS WHICH ARE UNKNOWNS

EXPLANATION OF TABLE - SYMBOLS AT THE START OF EACH ROW INDICATE THE UNKNOWN PARAMETER TO WHICH

$$\begin{bmatrix}
\phi_1 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\
 & \lambda_1 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 & 20 & 21 \\
 & & \phi_2 & 22 & 23 & 24 & 25 & 26 & 27 & 28 & 29 & 30 \\
 & & & \lambda_2 & 31 & 32 & 33 & 34 & 35 & 36 & 37 & 38 \\
 & & & & \phi_3 & 39 & 40 & 41 & 42 & 43 & 44 & 45 \\
 & & & & & \lambda_4 & 46 & 47 & 48 & 49 & 50 & 51 \\
 & & & & & & \phi_4 & 52 & 53 & 54 & 55 & 56 \\
 & & & & & & & \lambda_5 & 57 & 58 & 59 & 60 \\
 & & & & & & & & \phi_5 & 61 & 62 & 63 \\
 & & & & & & & & & \lambda_5 & 64 & 65 \\
 & & & & & & & & & & \phi_6 & 66 \\
 & & & & & & & & & & & \lambda_6
\end{bmatrix}$$

Table 4.6

THE MISCLOSURE
(USER REQUIRED VARIANCE - VARIANCE COMPUTED FROM APPROXIMATE
V/C MATRIX FOR PARAMETERS)

TYPE KEY- A INDICATES AN AZIMUTH
(UNITS FOR VARIANCE ARE ARC SECONDS SQUARED)

G INDICATES AN ANGLE
(UNITS FOR VARIANCE ARE ARC SECONDS SQUARED)

S INDICATES A DISTANCE
(UNITS FOR VARIANCE ARE METERS SQUARED)

ESTIMABLE NUMBER	ESTIMABLE TYPE	W C	STATIONS INVOLVED	ESTIMABLE TYPE	W C
1	G	0.00E+00	17	S	-1.15E-02
2	G	0.00E+00	18	S	-1.21E-02
3	G	-9.83E+00	19	S	-8.96E-03
4	G	-1.38E+01	20	S	-1.85E-02
5	G	0.00E+00	21	S	-1.86E-02
6	G	-2.27E+00	22	S	-1.36E-02
7	G	-2.04E+01	23	S	-1.71E-02
8	G	-1.92E+01	24	S	-1.89E-02
9	G	-3.26E+01	25	S	-1.81E-02
10	G	-2.95E+01	26	S	-1.82E-02
11	G	-8.84E-02	27	S	-1.58E-02
12	G	-1.05E+00			
13	G	-1.37E+00			
14	G	-5.91E+00			
15	G	-5.35E+00			
16	G	-3.18E-01			
17	G	-1.15E-02			

Table 4.7

SOLUTION FOR THE OFF-DIAGONAL COEFFICIENTS IN THE V/C MATRIX OF PARAMETERS (X_C)

COEFFICIENT NUMBER	A SOLUTION VALUE	B SOLUTION VALUE
1	-2.4988403E-06	-3.3467368E-06
2	1.2372038E-06	2.6537600E-06
3	-3.0985830E-06	-6.1003175E-06
4	-6.7330438E-07	-5.5092096E-07
5	-2.4882866E-06	-1.8952287E-06
6	1.1617922E-06	2.4260225E-06
7	4.1334260E-07	-8.7230001E-07
8	5.9223239E-06	5.3556860E-06
9	8.7048102E-07	4.4266553E-06
10	-2.4043666E-06	-1.4180550E-06
11	6.8014924E-06	7.7869136E-06
12	-4.3517357E-06	-1.3047975E-06
13	7.9316524E-06	8.4831554E-06
14	-1.2656983E-07	2.1992992E-06
15	4.1690600E-07	5.2371888E-07
16	-4.3458385E-07	2.8040085E-07
17	7.5413845E-07	5.0325616E-06
18	5.6771541E-06	1.0443910E-06
19	2.8490176E-06	6.4007472E-06
20	1.6100967E-06	9.5537689E-07
21	5.3106633E-06	5.0250583E-06
22	3.8196304E-06	1.8165701E-06
23	1.0540155E-05	9.9400913E-06
24	-3.1356558E-07	-3.1466607E-07
25	1.3324179E-05	9.8974560E-06
26	-3.3507240E-06	-1.2648343E-08
27	6.6419016E-06	5.8848009E-06
28	2.9712683E-06	8.9766399E-07
29	9.8640448E-06	8.9023088E-06
30	1.2241944E-06	-1.0613003E-06
31	5.1159568E-06	3.4164941E-06
32	1.3933546E-05	1.4537814E-05
33	-3.3270135E-06	1.3904000E-06
34	8.5635756E-06	1.5210004E-05
35	-4.0273590E-06	-4.0007317E-06

Table 4.8

SOLUTION FOR THE OFF-DIAGONAL COEFFICIENTS
(CONTINUED)

COEFFICIENT NUMBER	A SOLUTION VALUE	B SOLUTION VALUE
36	-3.4126715E-06	-3.0630326E-06
37	1.5406258E-06	3.4955192E-06
38	5.1298994E-06	7.6401629E-06
39	-6.7325618E-07	2.6823545E-07
40	3.1849213E-06	9.6291951E-06
41	-2.5381541E-06	3.5225821E-07
42	0.0	3.2080570E-06
43	0.0	-2.9396615E-06
44	9.4465458E-06	9.3211884E-06
45	-1.7784478E-06	-3.2997195E-06
46	8.3764280E-07	6.1237006E-07
47	1.5355792E-05	1.6343707E-05
48	0.0	1.4692614E-07
49	0.0	-1.1772399E-06
50	2.6729977E-06	1.2331502E-06
51	1.2356424E-05	1.3170270E-05
52	4.0712184E-06	5.1341158E-07
53	0.0	5.6399758E-06
54	0.0	-1.8401915E-06
55	1.2245568E-05	8.9552195E-06
56	-1.1482189E-06	-9.5654832E-07
57	0.0	-1.0099902E-06
58	0.0	2.4939218E-06
59	1.3861863E-06	1.0475487E-06
60	9.0053845E-06	1.3860612E-05
61	8.6158980E-07	3.6998681E-06
62	8.9103269E-06	7.7212317E-06
63	-2.5098579E-06	1.1819247E-07
64	-4.7011144E-06	-4.2294705E-06
65	6.4226915E-06	8.0710452E-06
66	-2.5088375E-06	-2.4965702E-06

Table 4.8 (Continued)

PREDICTED VARIANCES FOR ESTIMABLES
USING CRITERIAN V/C MATRIX OF PARAMETERS

ESTIMABLE NUMBER	A SOLUTION ESTIMABLE VARIANCE	B SOLUTION ESTIMABLE VARIANCE
1	6.67E-01	3.26E-01
2	1.14E-01	7.58E-02
3	8.91E-01	6.40E-01
4	8.86E-01	6.56E-01
5	8.32E-01	5.68E-01
6	9.35E-01	4.67E-01
7	8.90E-01	7.66E-01
8	8.90E-01	7.29E-01
9	8.89E-01	7.22E-01
10	8.92E-01	7.95E-01
11	8.96E-01	7.33E-01
12	8.83E-01	8.00E-01
13	8.45E-01	3.55E-01
14	8.90E-01	6.26E-01
15	8.85E-01	6.26E-01
16	9.00E-01	8.12E-01
17	8.75E-03	8.88E-03
18	7.94E-03	7.94E-03
19	1.12E-02	1.12E-02
20	1.67E-03	1.96E-02
21	1.43E-03	1.62E-3
22	6.41E-03	6.46E-03
23	3.01E-03	2.87E-03
24	1.28E-03	1.36E-03
25	1.86E-03	2.24E-03
26	1.84E-03	1.88E-03
27	4.24E-03	4.39E-03

Table 4.9/10

THE MISCLOSURE
(USER REQUIRED VARIANCE - VARIANCE COMPUTED FROM APPROXIMATE V/C MATRIX FOR PARAMETERS)

TYPE KEY- A INDICATES AN AZIMUTH
(UNITS FOR VARIANCE ARE ARC SECONDS SQUARED)

G INDICATES AN ANGLE
(UNITS FOR VARIANCE ARE ARC SECONDS SQUARED)

S INDICATES A DISTANCE
(UNITS FOR VARIANCE ARE METERS SQUARED)

ESTIMABLE NUMBER	ESTIMABLE TYPE	W C	STATIONS INVOLVED	ESTIMABLE TYPE	W C
1	G	0.00E+00	7	S	0.00E+00
2	G	0.00E+00			
3	G	-4.31E-01			
4	G	-1.18E+00			
5	G	0.00E+00			
6	G	0.00E+00			

Table 4.11

SOLUTION FOR THE OFF-DIAGONAL COEFFICIENTS
IN THE V/C MATRIX OF PARAMETERS (X_C)

COEFFICIENT NUMBER	A SOLUTION VALUE	B SOLUTION VALUE
1	8.7138119E-10	-4.9814091E-09
2	5.7217555E-09	-2.5464317E-08
3	1.1394299E-09	1.4065918E-08
4	-1.0592387E-08	-6.5748338E-08
5	5.4320424E-09	-5.2937963E-08
6	0.0	1.1675752E-07
7	0.0	1.0819686E-08
8	4.6110671E-09	-5.7127525E-10
9	1.6796129E-09	1.0146863E-08
10	1.4003971E-09	-3.5252736E-09
11	-9.1179544E-09	2.2839092E-08
12	4.2360853E-09	2.8921619E-08
13	2.9212543E-10	-4.9885916E-08
14	-1.6155957E-09	6.1287153E-08
15	8.2841778E-10	1.8280474E-08
16	0.0	-7.3818910E-08
17	0.0	1.2704849E-08
18	-3.3311949E-09	-7.9761051E-09
19	-6.9263706E-10	-2.2353053E-09
20	-1.7405088E-10	-3.4768266E-09
21	1.1323913E-09	2.2602833E-08
22	4.3517186E-08	4.3026773E-09
23	9.1440810E-10	-9.8512452E-09
24	-4.4134566E-08	-9.4516679E-08
25	6.6404368E-08	1.3626459E-07
26	-1.5607224E-08	6.6083636E-09
27	2.7786200E-09	-3.2725840E-08
28	2.8677505E-10	3.3504275E-08
29	-1.7982658E-09	-3.2436107E-09
30	1.1708941E-08	2.1112868E-08
31	1.1320748E-07	4.2995708E-08
32	4.8616428E-08	5.6730443E-08
33	-1.6598824E-07	-1.0321617E-07
34	4.8599752E-08	-1.8014305E-08
35	5.2947762E-09	3.9400810E-08

Table 4.12

SOLUTION FOR THE OFF-DIAGONAL COEFFICIENTS
(CONTINUED)

COEFFICIENT NUMBER	A SOLUTION VALUE	B SOLUTION VALUE
36	1.7163018E-09	-6.6579418E-09
37	2.7796043E-09	1.9205513E-09
38	-1.8091217E-08	-1.2471553E-08
39	-3.7594518E-08	-1.4936165E-07
40	2.5167299E-07	4.3515365E-07
41	-7.3997001E-08	-3.6176925E-08
42	0.0	-5.1839098E-08
43	0.0	3.5642870E-08
44	0.0	-7.0065802E-09
45	0.0	4.5631317E-08
46	7.6348215E-08	3.5515620E-07
47	-1.8719192E-08	-8.1052072E-09
48	0.0	-4.8199357E-08
49	0.0	4.5808186E-08
50	0.0	-9.7824149E-09
51	0.0	6.3716868E-08
52	8.9640082E-08	1.2284989E-08
53	0.0	1.0420212E-07
54	0.0	-8.5367674E-08
55	0.0	1.6146771E-08
56	0.0	-1.0502265E-07
57	0.0	-3.1008929E-10
58	0.0	-1.0776382E-08
59	0.0	6.8929467E-09
60	0.0	-4.4833040E-08
61	-1.9697382E-09	1.5525796E-09
62	0.0	-2.3837010E-09
63	0.0	1.5519696E-08
64	0.0	4.4606026E-09
65	0.0	-2.8962461E-08
66	-2.6060474E-09	1.6262106E-13

Table 4.12 (Continued)

PREDICTED VARIANCES FOR ESTIMABLES
USING CRITERIAN V/C MATRIX OF PARAMETERS

ESTIMABLE NUMBER	A SOLUTION ESTIMABLE VARIANCE	B SOLUTION ESTIMABLE VARIANCE
1	3.89E-02	3.98E-02
2	6.64E-01	6.83E-01
3	8.90E-01	3.21E-01
4	8.90E-01	5.30E-01
5	6.06E-02	6.00E-02
6	1.40E-01	1.44E-01
7	1.68E-03	1.24E-03

Table 4.13

Chapter 5. Choice of Observations Needed to Establish the Network

5.1 Introduction.

In Chapter 4, the variance-covariance matrix for the positional parameters was formed only from user requirements and NOT from the observations which usually establish the parameters. The only statement that can be made about this V/C matrix ($\Sigma_x(CRIT)$) are that it does fulfill user requirements and that it is structured in the same manner as other V/C matrices. Whether this Σ_x matrix could be the result of observations of the type the designer can have made to establish the network is not immediately obvious. By itself, $\Sigma_x(CRIT)$ means nothing. In this chapter, a use for $\Sigma_x(CRIT)$ will be suggested and some examples presented where this suggested use has been implemented in a design problem.

5.1.1 Discussion of the design procedures.

At this point in the design process, there exists a $\Sigma_x(CRIT)$ matrix which fulfills the user requirements for the network to be designed. This $\Sigma_x(CRIT)$ is independent of the resources allocated to perform the establishing survey and should be thought of as a yard stick to measure the effectiveness of various design configurations. The goal is not to create a Σ_x from allocated observations which is "better" than the $\Sigma_x(CRIT)$, in the sense defined by Federov, (1972), since one

which is uniformly better by the Federov criteria, one of which is given below, may be prohibitively expensive. The final product of the design should be selectively as good as or better than $\Sigma_x(\mathrm{CRIT})$ matrix with respect to those quantities the user is interested in only. Thus, those quantities which the user requires are the key to the choice of observations. They may be station position accuracies and accuracies of quantities which are functions of station position (the estimables).

The most physically meaningful of the Federov criteria is the test referring to the configurations of the criterian and design hyperellipsoids. The design is better, using this rule, if

$$\underset{u\,u}{Q} = \Sigma_x(\mathrm{CRIT}) - \Sigma_x(\mathrm{gen})$$

and Q is a positive semi-definite matrix.

Let us suppose that in the above equations, $\Sigma_x(\mathrm{gen})$ represents the result of a design selection of observations which satisfies all user requirements. In all the experiments performed as a part of this study this difference (Q) matrix between criterian and design V/C matrices was neither positive or negative semi-definite, which was interpreted to mean that the design was neither better nor worse than the criterian V/C matrix and that other tests must be devised to differentiate between criterian and design V/C matrices. As indicated above, the user has already supplied the means to make these tests when his requirements are quantified. Once we have a $\Sigma_x(\mathrm{gen})$ which is satisfactory, the question should be asked of the user, who is paying for the control establishment either

directly or indirectly, as to whether the increased cost required to make the above Federov test Q positive semi-definite is justified. What additional benefit is gained by doing this addition in these days of cost consciousness? Any additional effort expended to that end may be very difficult to justify.

One question yet to be answered is the role of the designer himself will play in the process. The implication of Bossler, et al (1973), is that a matrix, Z, which is apparently not unique, can be solved for in such a manner that:

$$\Sigma_{L_b}^{-1} = A^{-\prime} \Sigma_x^{+} A^{-} + Z - (AA^{-})^{\prime} Z AA^{-} \tag{5.1}$$

$A^{-} \equiv$ a generalized inverse of A and where $\Sigma_{L_b}^{-1}$ is a positive semi-definite diagonal matrix. While this may be mathematically correct, the feasibility of a Σ_{L_b} matrix such that $P \equiv \Sigma_{L_b}^{-1}$ may be beyond the allocation of the effort for the project. Since the designer is familiar with both the feasibility and cost of execution of any design, the process of observation selection should take advantage of this familiarity. If either of two observations may improve a design so that it meets user requirements, there is usually a "better" (more economical) observation to include when considerations of equipment, station occupation, project phasing with respect to time and available manpower and expertise are all considered. The algorithms to account for these considerations are complex, so translation into mathematical equations for inclusion in the design process, is at best, a difficult task. The designer, however, is already programmed to make this type

decision. Even with a minimum of training the designer is usually qualified to appreciate which observation is better to use. The design process developed in this chapter takes advantage of this designer capability. The solution for eq (5.1) will be discussed in detail in Chapter 6.

In all the design examples, a minimum constraint solution is the design objective. As was previously stated, the estimable variances in this case represent an inherent uncertainty in an estimable which results form the inherent uncertainties of the establishing observations themselves. In the sense that any overconstrained solution represents a situation where the confidence in the determination of station coordinates may be overstated, these minimum constraint solutions are a worst case analysis of the estimable variances. For this reason, the designer should always plan for this worst case first. In this manner, even if the overconstraint is incorrect (that is, the station coordinates of the constrained stations are not known a priori as well as the designer first thinks) the user requirements will still be met. The last preliminary point to be considered in observation selection is the choice of types of observation. By the examination of the estimables, this choice can be made by either employing the word or the mathematical definitions discussed in Chapter 2. If there is any doubt as to the estimability of a user required quantity, it would be prudent to implement the testing procedure which is also outlined in that same chapter.

5.2 **Empirical investigation of the variance-covariance matrix of all possible observations.**

In this discussion, the design matrix, A , will refer to all observations of any given type that the designer would possibly use (or those types which could be used). These should be selected on the basis of:

1. the estimables,
2. the cost of each observation type,
3. reconnaissance reports on the feasibility of different observation types.

Further, this A matrix will be scaled by $P^{1/2}$, which is defined such that

$$P^{1/2} P^{1/2} = \Sigma_{L_b}^{-1}$$

$$A = P^{1/2} \times \text{original design matrix} = P^{1/2} A_o$$

and, since in all cases discussed, the observations are assumed to be independent, this $P^{1/2}$ matrix simplifies to a diagonal matrix whose major diagonal elements are the reciprocals of the standard deviations of each of the possible observations. Thus

$$\underset{u\,n}{(A_o'} \, \underset{n\,n}{\Sigma_{L_b}^{-1}} \, \underset{n\,u}{A_o)} \, \underset{u\,1}{X} + \underset{u\,n}{A_o'} \, \underset{n\,n}{\Sigma_{L_b}} \, \underset{n\,1}{W} = [\underset{n\,n\,n\,u}{(P^{1/2}A_o)'} \, \underset{n\,n\,n\,u}{P^{1/2}A_o}] \, \underset{u\,1}{X} + \underset{n\,n\,n\,u}{(P^{1/2}A_o)'} \, \underset{n\,n\,n\,1}{P^{1/2}W}$$

$$= 0$$

where A_o is as defined in section 2.1.

Mathematically, this transforms the model to the classic Gauss-Markov case, where the normal equations are simply $(A'A)$. Note that n in this instance is the maximum number of possible observation.

If the V/C matrix of the predicted values of all possible observations of this type (that is, scaled observations) is formed, based on Σ_x, it will be an $n \times n$ symmetric positive semi-definite matrix, Σ_{Lp}.

Physically, the scaling equalizes the weight of each observation represented by a row in the A matrix, regardless of type.

If, for a given problem, Σ_x is formed from some of the rows of the matrix (that is, some of the possible observations) and then Σ_{Lp} is computed, intuitively it would be expected that this fact should be discernable from the structure of the Σ_{Lp} matrix. The steps in the procedure for testing this proposition was as follows:

1. Define from the estimables what types of observations will be required.
2. Form $A'A$ from all or some of each type required so that some redundancy in measurement exists.
3. Compute Σ_x using the pseudo-inverse after deciding geometrically what the rank deficiency is in the free normals.
4. Form Σ_{Lp} from all the scaled observations in the A matrix. That is, treat all possible observations allocated to the establishment by choice of observation types as estimables themselves, predictable from the parameters, and form the V/C

matrix for these observation/estimable quantities,

$$\Sigma_{Lp} = A \Sigma_x A'$$

5. Compute the correlation coefficient matrix corresponding to $A \Sigma_x A'$

Tests were performed utilizing the above procedure on a variety of networks with very different configurations. The only points these networks had in common were that none had more than 14 points included and all had distances of such length that the Gaussian Mid-latitude and design matrix coefficient expressions given in Chapter 2 were adequate approximations. The characteristics which appear to be common to all such correlation coefficient matrices are listed below. In this list of characteristics, observations which actually defined the V/C matrix of parameters (that is, were included in the design matrix which formed Σ_x and Σ_{Lp}) are called "used observations". Observations which were possible to make but were excluded from the design matrix are called "unused observations". These common characteristics are:

1. Observations (that is, scaled observations) not included in the formation of the Σ_x matrix (unused observations) appear to have generally higher positive correlations than do those observations which formed Σ_x (used observations).

2. Unused observations are positively correlated to both other unused and some used observations, regardless of types.

3. Some used observations are highly positively correlated to other used and some unused observations.

4. In networks with reasonable degrees of freedom for the size (df > 5 to 10 for networks where the number of stations is between 7 and 15), the deletion of an unused observation has no effect on the Σ_x matrix and the deletion of a used observation which has a high positive correlation with respect to one or more other used observations has a smaller effect on the Σ_x matrix than the deletion of a used observation with relatively low correlation ($\rho < .5$).

5.2.1 An illustration of the effect of the removal of correlated and uncorrelated observations from the formation of the free normals and pseudo-inverse.

The network chosen for this example is T3. Table 5.2.1 indicates which observations were used. In this case the estimables chosen were angles and distances, so directions and distances were the measured quantities and the free normals are rank deficient by three. The effect of removal on the estimables is also illustrated for each of the three tests. Station variances refer to the pseudo-inverse type constraint.

Tables 5.2.2 and 5.2.3 are the correlation coefficients from the $\Sigma_{L\rho}$ matrix of "used observations" and of all possible observations respectively. Note the characteristically higher positive correlations occurring in those observations in 5.2.3 which were not used in Σ_x. In Tables 5.2.4 and 5.2.5 the column marked "ALL*OBS" gives the variances of positional parameters and estimables from the Σ_x established as indicated using the "* obs" in Table 5.2.1.

When observation number 26 (direction 6-8) (see Figure 5.1) is removed from the design matrix, the matrix exhibits a change in the second place in the variance of latitude and longitude for several parameters and a change in the first place in variances for two parameters, the largest being $0.''^2 2\times10^{-5}$ for the latitude of station 8. For the estimables, 13 of 48 changed in the second place, 4 of 48 changed in the first place. The largest estimable angle change was in #20 of $0.''^2 2$ and the largest distance variance change occurred in #46, 0.2×10^{-2} meters. With the removal of direction 4-6, observation #16, seven station coordinate variances (in latitude and longitude) changed in the second place, while one changed in the first place by $0.''^2 1\times10^{-6}$. Seven of the estimable variances changed in the second place. The largest estimable angle variance change was $0.''^2 5$ in estimables 11 and 12. The largest distance estimable variance change was 0.5×10^{-3} m in estimables number 36 and 37, see Tables 5.2.4/5.

With the removal of the uncorrelated observation, direction 5-3, #18 (correlated with other observations less than .5), five station coordinate variances changed in the second place, six changed in the first place, the largest being $0.''^2 1\times10^{-5}$ in the variance for the latitude of station 3. For the estimables, ten variances changed in the second place. The largest change in an estimable angle was for #16. For the distances, the largest change was 0.15×10^{-2} m for #37, see Tables 5.2.4/5.

While the preceeding test and those others performed to test this assumption of dependence indicate that , as a general rule, the assumption

is correct, one important fact should be kept in mind. The correlation between any row in the scaled A matrix and any other row is a function of the coefficients in each of the rows and the Σ_x matrix. These correlations are unchanged, even if some of the other rows of the A matrix are removed from the matrix. What this implies is that correlations exist between predicted observations even when not enough observations are considered to form a given Σ_x matrix. For this reason, the widest possible selection of observations should be included in the scaled A matrix. If, because of this inclusion, an uneconomical first design is made, the process of selection can be iterated, as illustrated in design example 5.8.1, and discussed in section 5.7.

5.3 Interpretation of the empirical results.

If the characteristics enumerated in 5.2 are interpreted in the light of the intuitive statement made by Hirvonen (1965), the high positive correlations between rows in the scaled A matrix indicate a dependence between the quantities described by each row. Since these are the predicted values of the quantities which are actually observed, it would be expected that more information would be gained by observing uncorrelated (independent) quantities than those which are predicted to be dependent by the mathematical model.

If this is correct, the substitution of a $\Sigma_x(CRIT)$, derived as described in Chapter 4, for the $\Sigma_x(gen)$ (generated from a sample set of actual scaled observations) should also be interpretable in the same manner. In subsequent examples this is exactly what will be done.

5.4 Assumptions made in experiments for observations.

5.4.1 A priori standard deviation for observations.

The three most common types of observations used in network establishment are directions, distances and azimuths. These three will be used in the following examples of design, but should not be considered as all inclusive. The inclusion or substitution of other observation types can be handled in exactly the same manner as the three chosen.

The following are the a priori standard deviations for each type of observations:

direction -	$\sigma = \frac{1.''6}{\sqrt{n}}$	(where n is the number of plate positions observed)
distance -	$\sigma = 0.^{m}017 + S^{m}/10^{6}$	(where S is the distance between the points in meters)
azimuth -	$\sigma = 0.''45$	(first order astronomic requirements from (USDC))

5.4.2 Order of preference for selection of observations.

Both distances and directions are treated with equal preference in the elimination of observations due to their correlation in the Σ_{LP} matrix. Azimuths, where included in the A matrix, were given first priority for removal, since it is assumed that azimuths are more expensive to perform than either distance or directions.

5.4.3 Inclusion of the station unknowns, z , in the $\Sigma_x(CRIT)$ matrix.

In Chapter 4, no mention was made of the station unknown, z , in the Σ_x design criteria. The reason for this omission is that none of the estimables discussed was a function of any of the net station unknowns. If the diagonal term of the $\Sigma_x|_{X_{c^o}}$ matrix corresponding to the variance of the station unknown is set at some reasonable value, for example $0\overset{''}{.}5^2$, since the station unknown is referenced nowhere in the equation set 4.4, the $X'_c X_c$ contribution of the off diagonal terms in the z rows of the Σ_x matrix is minimum when these off diagonal terms are set to zero. In this chapter, when $\Sigma_x(CRIT)$ is used in conjunction with the direction observation equations, eq (2.10) through (2.12), the z rows are modelled to reflect this conclusion. That is:

$$COV(z_k, q_j) = 0$$
$$COV(z_k, q_k) = 0$$
$$COV(z_k, z_k) = 0\overset{''2}{.}50$$

where q_j is any ϕ , λ or the z of any other station, j , except when $z_k = q_k$

5.5 Degrees of freedom in test solutions.

Test solutions enforcing user requirements on station variances and estimable quantities are presented for different sets of estimable quantities and networks. In many of these solutions, degrees of freedom are fairly low. This should not be interpreted as advocating little or no redundancy. Additional degrees of freedom resulting from inclusion of additional observations of comparable accuracy to those used in the net establishment have three main effects. The first and most obvious

one is that they improve the definition of the parameters and derived estimables. The second effect is that the additional observations increase the confidence in the solution. That is, presuming that the variances for two solutions are comparable, the solution with the higher number of degrees of freedom has smaller confidence intervals for all determined quantities. The third, and probably the most important, effect is that additional judiciously placed additional observations will assist in the actual reduction of data from observations which are performed according to the designer's plan in that they will strengthen the network so that observational blunders may be more easily detected, Uotila (1974a).

On the other hand, if specifications do not include confidence intervals and additional observations add additional cost, some trade off must be made in a practical situation. The designs shown in the examples that follow are the minimum a survey party can return with and still meet requirements. If it is feasible for additional observations to be made, the common practice of performing them should be continued. For example, if the design calls for two directions from a station which sees a number of other occupied stations and these other occupied stations can show a light or target with little additional effort, certainly these non-required directions should be measured. If however, the non-required stations are unoccupied, the additional effort/cost to target stations may not be worth the benefit derived from the additional degrees of freedom gained.

5.6 Suggested scheme for selection of observations to be used.

Once the Σ_{L_P} matrix of all possible observations is formed and the correlation coefficients computed, selected observations will be correlated to others within the correlation coefficient matrix. Experimentally, a correlation coefficient of $\rho \geq .7$ seems to indicate that, for two different observations, one may be selected and the other (or others in the case of multiple correlations of one observation to more than one other) deleted. This cutoff point seems to produce a $\Sigma_X(gen)$ matrix which meets or betters user requirements in all cases tested, except those where a nominal target positional variance was assumed. However, this often produced economically expensive networks as well. For example, observed azimuths do not only orient the network but also determine station coordinates and such estimables as angles to very good accuracies, due to the small standard deviation assigned the observations. A very well determined net always results when many azimuths of this type are included. The mechanical exclusion of observations with high correlations recognizes this fact and often produced networks with many azimuths and few directions. These networks are successful in that they meet user requirements but are prohibitively expensive.

To get a practical and inexpensive design, a much lower cutoff in correlation coefficient is suggested, something of the order of $\rho \geq .55$. Also the following order of elimination is suggested:

Correlated to Observation Type

Type	Direction	Distance	Azimuth
direction	delete either	delete distance	delete azimuth
distance	delete distance	delete either	delete azimuth
azimuth	delete azimuth	delete azimuth	delete either

(Note that the above table applies to the observations of a type and accuracy used for this study and their individual "cost" only. There may be a considerable change in this order of preference if, for example, gyro-theodolites are available. These would give azimuths of considerably poorer accuracy which would still have the effect of orientation of the network, and this orientation could be gained at considerable less cost than performance of first order astronomic azimuth observations.)

There are two added requirements to the above scheme.

1. the degrees of freedom must be at least one
2. at least one of each type of observation required to make the target quantities estimable must be included.

A further groundrule is added in the illustrated examples. Since the station unknown, z , is included in the mathematical model for each station, at least two directions must be included for each station. If a test is being made where it seems from the correlation coefficients that a station need not be occupied by a direction measuring party, a very low weight may be assigned to any two directions,

effectively removing any effect they would have on the accuracy of the coordinate and estimable accuracy determination.

This solution usually fails to entirely satisfy user requirements. Additional observations may be added iteratively, to improve the accuracies of the design. From a cost basis, the following order was used in the examples:

1. Additional (that is, unused direction) observations from occupied stations.
2. Additional (that is, unused) observations of distances.
3. Additional azimuths, added only as a last resort, for economic reasons.

The choice of these additional observations introduces some subjectivity to the design. This capitalizes on the experience of the designer who can usually select additional observations from stations which add minimum cost to the overall project. Generally, any observation involving a station or in an area where criteria have not been met will improve the accuracies of quantities involving that station or in that area.

The question of when to stop these additions should be answered by the designer, who has an understanding of the use for which the net is intended. Is a variance of 0.2 sufficient if the target variance is 0.16? Is 0.2 substantially different from 0.16? In this study, the choice was made to stop iterative addition of observations when the predicted variance of any estimable was no more than 5 units in the second significant figure greater than the target value for the variance of

that estimable. In the case of parameters, if the user placed a maximum magnitude on the accuracy of any position in some minimum constraint network, iteration was stopped when this was achieved, so long as other user requirements were met as well. If the position accuracy was nominal (that is, not of vital importance as compared to accuracies of estimables) iterative addition was stopped after estimable target variances were met or bettered.

5.7 Iteration of the design procedure.

If, after the process of initial selection and sequential addition of observations, a design is made which meets user requirements, but is too expensive in the user's/designer's opinion, this entire process may be repeated, substituting the $\Sigma_x(\text{gen})$ from the initial solution for the $\Sigma_x(\text{CRIT})$ from Chapter 4. $\Sigma_x(\text{gen})$ matrix has the virtue of being a physically feasible one, using allocated observation types. As previously noted, $\Sigma_x(\text{CRIT})$ is independent of this consideration. The success of this procedure with $\Sigma_x(\text{CRIT})$ seems to depend on how mutually attainable the users requirements for estimables and station positions are. If the user requirements are mismatched, for example, a required station accuracy of 5 meters in latitude and longitude and a required estimable azimuth accuracy of $0.''4$, in the actual result will most probably determine stations to much better accuracy than 5 meters to attain the azimuth requirement. Since the Σ_{L_b} used in the formation of $A_0'\Sigma_{L_b}^{-1}A_0$ and therefore $(A_0'\Sigma_{L_b}^{-1}A_0)^+$ is not unique, it can be inferred that this

procedure can be iterated to form a different set of used observations and, therefore, a network which may cost less, (Bossler et al (1973)).

5.8.1 Design example 1, test net T3.

Using the $\Sigma_X(CRIT)$ derived in section 4.8.1 with directions, distances and azimuths, the type observation standard deviations are as follows:

directions - $\sigma = 0.''40$ per direction set (16 positions)

distances - $\sigma = 0.^{m}017 + 5^{m}/10^{6}$

azimuths - $\sigma = 0.''45$

The cutoff for initial selection of observations in this case was $\rho \geq .55$.

Table 5.8.1 gives the allocated observations and Table 5.8.1-2 indicates the user designated required estimables and their target variances. The station variance selection, quantification of user requirements and the configuration of $\Sigma_X (CRIT)$, the criterian V/C matrix are all shown in Section 4.8.1. Table 5.8.1-3 is the correlation dependence developed from the $A\Sigma_X(CRIT)A'$ matrix, which will be used to make the initial selection of observations from Table 5.8.1-1. Table 5.8.1-4 is the initial selection of observations along with subsequent iterations where observations were added to meet user requirements. Table 5.8.1-5 indicates the resulting station and estimable variances for each selection of observations. That portion of the table marked final represents the design for the network, which is shown graphically

in Figure 5.8.1-1. The "minimum constraint" columns of Table 5.8.1-5 indicate the solution for this final design, fixing station 1 in ϕ, λ; note the invariance of the estimable variances. This problem was iterated as described in Section 5.7. Table 5.8.1-6 are the correlations developed from the $A \Sigma_x(gen) A'$ matrix based on the $\Sigma_x(gen)$ developed in the previous selection process. Note the extreme correlations in that section of the table developed for the azimuth observations possible (rows 39 through 57). Table 5.8.1-7 represents the initial selection of observations as well as the iterations due to addition of observations. Table 5.8.1-8 gives the station coordinate variances as well as those of the estimables for each selection. That section of the table marked final is the design adopted from this process, graphically presented in Figure 5.8.1-2.

The results of one other than pseudo-inverse minimum constraint, that of fixing station one, is given in the last column of Table 5.8.1-8.

5.8.2 Example 2, test net T4.

The example uses the $\Sigma_x(CRIT)$ matrix developed in Section 4.8.2. The variances of the station coordinates, the user requirements, the configuration of the $\Sigma_x(CRIT)$ matrix and the predicted variances of the estimables are given in that section. The allocated observations and the target estimable variances are given in Tables 5.8.2-1 and -2 respectively. Table 5.8.2-3 shows the correlations developed in the $A \Sigma_x(CRIT) A'$ matrix, upon which the initial design will be based. This initial design is given in Table 5.8.2-4 as well as the subsequent

iterations where additional observations are added to meet user requirements. The section that is marked final of Tables 5.8.2-5 and -6 is the design selected. Figure 5.8.2-1 graphically indicates the final design configuration. Table 5.8.2-6 indicates the variances of a minimum constraint solution other than the pseudo-inverse. Note the invariance of the estimable variances in this solution as compared to that marked <u>final</u>.

Tables 5.8.2-7 and -8 show the effect of an overconstraint on the network. Again, it points up the minimum constraint as a worst case analysis.

5.8.3 Example 3, test net T3.

The user requires that the stations be determined to an accuracy better than a quarter of a meter (standard deviation) in latitude and longitude. The extimables required are angles and distances.

The accuracy adopted for the major diagonal elements of the Σ_x(CRIT) matrix is (5 cm)^2 at the mean latitude of the net using the rule of thumb given in Chapter 2. This translates to:

$$\Sigma_{x\,\phi_i\phi_i} \sim 0.''^2\,16 \times 10^{-5}$$

$$\Sigma_{x\,\lambda_i\lambda_i} \sim 0.''^2\,28 \times 10^{-5}$$

The three sigma criterian, as discussed in Chapter 3, is applied to the estimables. A c value of 5% is applied to the angles and one of 15% applied to the distances from Table 3.1, column two ($C\,(36_0)$).

Thus:

$$\sigma'^2_{ANGLE} = 0.''^2 64 \times 1.33 \simeq 0.''^2 7$$

$$\sigma'^2_{DISTANCE} = 3.40\ \sigma^2_{0\,DISTANCE}$$

The estimables to which this applies are listed in Table 5.8.3-2.

The allocated observation are directions consisting of 16 position observation sets, each of which has a standard deviation of $\frac{1.''6}{\sqrt{16}} = 0.''4$. Distances will be measured with an instrument whose accuracy is comparable with that discussed in Section 5.4.1. Table 5.8.3-1 indicates all possible observations of the above types from initial reconnaissance.

Thus:

$$\sigma'^2_{DIRECTION} = 0.''^2 16$$

$$\sigma'^2_{DISTANCE} = (0.^m017 + 5^m/10^6)^2$$

The following are decoupled (see Chapter 4) stations (note that if station i is decoupled from station j, it follows that j is decoupled from i, so the table below lists only the i-j combinations).

Station Number	Decoupled From
1	5,6,7,8,9
2	5,6,7,8,9
3	7,8,9
4	8,9
5	8,9

The configuration of the $\Sigma_X(CRIT)$ matrix is illustrated in Table 5.8.3-3. The A and B solutions are presented in Table 5.8.3-4. The predicted estimable variances based upon the $\Sigma_X(CRIT)$ matrix generated in the

it is very dependent upon the knowledge and experience of the designer. The procedure has the virtue that it is simple to apply to a variety of situations without very much change, as opposed to a mathematical optimization whose cost algorithms would require tailoring for each new problem. The disadvantage to the procedure is that no pre-selection of observations from the list of all possible observations is feasible and, due to the nature of the correlations, selection on the basis of correlation magnitude does not automatically guarantee that user requirements are met. Essentially, the procedure suggested in this chapter is one of trial and error after initial selections, based on "cost" and correlation, are made.

A and B solutions are given in Tables 5.8.3-5 and -6. Table 5.8.3-7 shows the relationships generated from the $A \Sigma_x(\mathrm{CRIT}) A'$ matrix where A is the scaled design matrix of all possible observations (that is, those given in Table 5.8.3-1). Observations selected are indicated in Table 5.8.3-8. The initial and subsequent determinations in terms of the station and estimable variances are given in Table 5.8.3-9. Those sections of the table marked final are the designer selected configuration. Figure 5.8.3-1 is the final observation choice indicated graphically. Table 5.8.3-10 represents two other minimum constraint solutions. Note that while the station variances are considerably different, the estimable variances remain the same as those from the pseudo-inverse solution.

The effect of overconstraint in this case is illustrated in Table 5.8.3-10. As previously stated, the pseudo-inverse (or any other minimum constrant) solution does represent a worst case analysis. The problem for the designer in this case will not be the design but the statistical testing on the weighted sum of the squares of the residuals from the observations made.

5.9 Summary

As can be seen from the examples, the choice of observations based on predicted correlations using the generated $\Sigma_x(\mathrm{CRIT})$ works quite well. In a sense, this is an optimizing procedure which uses the designer's grasp of the "cost" of a design to select observations which will keep the cost to reasonable levels. As an optimizing procedure,

DESCRIPTION OF THE OBSERVATIONS

KEY - D INDICATES A DIRECTION
(UNITS FOR VARIANCE ARE ARC SECONDS SQUARED)

A INDICATES AN AZIMUTH
(UNITS FOR VARIANCE ARE ARC SECONDS SQUARED)

S INDICATES A DISTANCE
(UNITS FOR VARIANCE ARE METERS SQUARED)

OBSERVATION NUMBER	TYPE	A PRIORI VARIANCE	FROM STATION	TO STATION
1*	D	0.160	1	2
2*	D	0.160	1	3
3*	D	0.160	1	4
4*	D	0.160	2	1
5*	D	0.160	2	3
6*	D	0.160	2	4
7*	D	0.160	3	1
8*	D	0.160	3	2
9*	D	0.160	3	4
10*	D	0.160	3	5
11*	D	0.160	3	6
12*	D	0.160	4	1
13*	D	0.160	4	2
14*	D	0.160	4	3
15*	D	0.160	4	5
16*	D	0.160	4	6
17*	D	0.160	4	7
18*	D	0.160	5	3
19*	D	0.160	5	4
20*	D	0.160	5	6
21*	D	0.160	5	7
22*	D	0.160	6	3
23*	D	0.160	6	4
24*	D	0.160	6	5
25*	D	0.160	6	7
26*	D	0.160	6	8
27*	D	0.160	6	9
28*	D	0.160	7	4
29*	D	0.160	7	5
30*	D	0.160	7	6

(NOTE - * INDICATES AN OBSERVATION USED TO FORM FREE NORMALS AND NORMAL PSEUDO-INVERSE)

Table 5.2.1

DESCRIPTION OF THE OBSERVATIONS
(CONTINUED)

OBSERVATION NUMBER	TYPE	A PRIORI VARIANCE	FROM STATION	TO STATION
31*	D	0.160	7	8
32*	D	0.160	7	9
33*	D	0.160	8	6
34*	D	0.160	8	7
35*	D	0.160	8	9
36*	D	0.160	9	6
37*	D	0.160	9	7
38*	D	0.160	9	8
39*	S	0.113X10**-2	1	2
40*	S	0.107X10**-2	8	9
41	S	0.118X10**-2	1	4
42	S	0.968X10**-3	2	3
43	S	0.205X10**-2	2	4
44	S	0.122X10**-2	3	4
45	S	0.109X10**-2	3	5
46	S	0.112X10**-2	3	6
47	S	0.866X10**-3	4	5
48	S	0.137X10**-2	4	6
49	S	0.117X10**-2	4	7
50	S	0.629X10**-3	5	6
51	S	0.836X10**-3	5	7
52	S	0.115X10**-2	6	7
53	S	0.158X10**-2	6	8
54	S	0.954X10**-3	6	9
55	S	0.937X10**-3	7	8
56	S	0.136X10**-2	7	9
57	S	0.116X10**-2	1	3

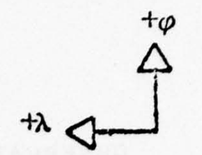
(NOTE - * INDICATES AN OBSERVATION USED TO FORM FREE NORMALS AND NORMAL PSEUDO-INVERSE)

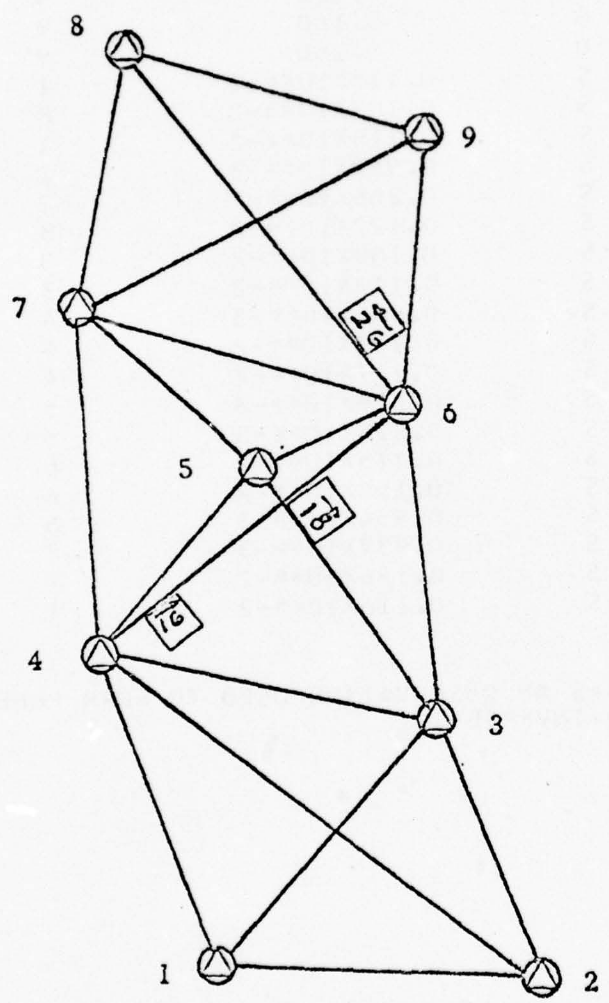
Table 5.2.1 (Continued)

NETWORK T3

LOCATION OF OBSERVATIONS

TO BE DELETED





Stations (△) joined by solid lines indicate that they are intervisible

Figure 5.1

ABSTRACT OF PREDICTED OBSERVATION
CORRELATION

EXPLANATION OF TABLE- COLUMN A IS THE OBSERVATION
BEING CONSIDERED

COLUMN B ARE THE OTHER OBSERVATIONS
WITH WHICH THE COLUMN A OBSERVATION
IS CORRELATED WITH COEFFICIENTS
OF BETWEEN 1. AND .80

COLUMN C ARE THE OTHER OBSERVATIONS
WITH WHICH THE COLUMN A OBSERVATION
IS CORRELATED WITH COEFFICIENTS
OF BETWEEN .79 AND .55

COLUMN A	COLUMN B	COLUMN C
*****	*************************	*************************
1	NONE	NONE
2	NONE	NONE
3	NONE	NONE
4	NONE	NONE
5	NONE	6
6	NONE	5
7	NONE	NONE
8	NONE	NONE
9	NONE	NONE
10	NONE	NONE
11	NONE	NONE
12	NONE	NONE
13	NONE	NONE
14	NONE	NONE
15	16	NONE
16	15	NONE
17	NONE	NONE

Table 5.2.2

ABSTRACT OF PREDICTED OBSERVATION
CORRELATION (CONTINUED)

COLUMN A	COLUMN B	COLUMN C
*****	************************	************************
18	NONE	NONE
19	NONE	NONE
20	NONE	24
21	NONE	NONE
22	NONE	NONE
23	NONE	NONE
24	NONE	20
25	NONE	NONE
26	31, 33, 34, 35, 38	37
27	36	NONE
28	NONE	NONE
29	NONE	NONE
30	NONE	NONE
31	26, 33, 34, 35, 38	37
32	NONE	37
33	26, 31, 34, 35, 38	37
34	26, 31, 33, 35, 38	37
35	26, 31, 33, 34, 38	37
36	27	NONE
37	NONE	32, 26, 31, 33, 34, 35, 38
38	26, 31, 33, 34, 35	37
39	NONE	NONE
40	NONE	NONE

Table 5.2.2 (Continued)

ABSTRACT OF PREDICTED OBSERVATION
CORRELATION

EXPLANATION OF TABLE- COLUMN A IS THE OBSERVATION BEING CONSIDERED

COLUMN B ARE THE OTHER OBSERVATIONS WITH WHICH THE COLUMN A OBSERVATION IS CORRELATED WITH COEFFICIENTS OF BETWEEN 1. AND .80

COLUMN C ARE THE OTHER OBSERVATIONS WITH WHICH THE COLUMN A OBSERVATION IS CORRELATED WITH COEFFICIENTS OF BETWEEN .79 AND .55

COLUMN A	COLUMN B	COLUMN C
1	NONE	NONE
2	NONE	NONE
3	NONE	NONE
4	NONE	NONE
5	NONE	6
6	NONE	5
7	NONE	NONE
8	NONE	NONE
9	NONE	NONE
10	NONE	NONE
11	NONE	NONE
12	NONE	NONE
13	NONE	NONE
14	NONE	45, 46
15	NONE	16
16	NONE	15

Table 5.2.3

ABSTRACT OF PREDICTED OBSERVATION
CORRELATION (CONTINUED)

COLUMN A	COLUMN B	COLUMN C
*****	************************	************************
17	NONE	NONE
18	NONE	NONE
19	NONE	NONE
20	NONE	24
21	NONE	NONE
22	NONE	NONE
23	NONE	NONE
24	NONE	20
25	NONE	NONE
26	31, 33, 34, 35, 38, 55	37
27	36	NONE
28	NONE	NONE
29	NONE	NONE
30	NONE	NONE
31	26, 33, 34, 35, 38, 55	37
32	NONE	37
33	26, 31, 34, 35, 38, 55	37
34	26, 31, 33, 35, 38, 55	37
36	27	NONE
37	NONE	26, 31, 33, 34, 35, 38, 55
38	26, 31, 33, 35, 55	57

Table 5.2.3 (Continued)

ABSTRACT OF PREDICTED OBSERVATION
CORRELATION (CONTINUED)

COLUMN A	COLUMN B	COLUMN C
*****	**************************	**************************
39	NONE	NONE
40	53	NONE
41	43	57, 44, 45, 46
42	NONE	43
43	41	44, 45, 57, 42, 46, 48
44	45, 46,	41, 43, 48, 57, 47, 49, 52
45	44, 46,	41, 43, 48, 14, 47, 49
46	44, 45	48, 14, 41, 43, 47, 49
47	48, 49	44, 45, 46
48	47	44, 45, 46, 49, 52, 43, 50
49	47	48, 51, 52, 44, 45, 46
50	NONE	48
51	52	49,54
52	51	48, 49, 56, 44, 54
53	40	NONE
54	NONE	568 518 52
55	26, 31, 33, 34, 35, 55	37
56	NONE	52, 54
57	NONE	41, 43, 44

Table 5.2.3 (Continued)

STATION VARIANCES
FOR THIS NET

KEY TO PARAMETER CODE- P INDICATES THE VARIANCE IN LATITUDE
L INDICATES THE VARIANCE IN LONGITUDE
Z INDICATES THE VARIANCE IN STATION
UNKNOWN ('Z' CORRECTION)

NOTE THAT ALL PARAMETER VARIANCES ARE IN UNITS OF ARC SECONDS SQUARED AND THE HEADING 'CODE' INDICATES PARAMETER CODE

STATION NUMBER/CODE		VARIANCE ALL *OBS	VARIANCE #26 DELETED	VARIANCE #16 DELETED	VARIANCE #18 DELETED
1	P	8.80D-06	8.98D-06	8.96D-06	9.21D-06
	L	1.30D-06	1.30D-06	1.30D-06	1.34D-06
	Z	3.44D-01	3.49D-01	3.51D-01	3.54D-01
2	P	9.08D-06	9.10D-06	9.47D-06	9.87D-06
	L	1.87D-06	1.90D-06	1.85D-06	1.96D-06
	Z	3.84D-01	3.86D-01	3.96D-01	3.92D-01
3	P	4.16D-06	4.20D-06	4.32D-06	5.12D-06
	L	1.77D-06	1.79D-06	1.80D-06	1.82D-06
	Z	1.71D-01	1.72D-01	1.75D-01	2.04D-01
4	P	1.97D-06	2.09D-06	2.03D-06	2.35D-06
	L	2.88D-06	2.94D-06	2.90D-06	2.86D-06
	Z	1.35D-01	1.39D-01	1.63D-01	1.35D-01
5	P	5.04D-07	5.21D-07	5.29D-07	5.83D-07
	L	1.46D-06	1.47D-06	1.48D-06	2.00D-06
	Z	1.54D-01	1.55D-01	1.56D-01	2.46D-01
6	P	8.77D-07	8.93D-07	9.73D-07	9.67D-07
	L	1.47D-06	1.51D-06	1.48D-06	1.52D-06
	Z	1.20D-01	1.36D-01	1.27D-01	1.27D-01
7	P	2.13D-06	2.17D-06	2.11D-06	2.16D-06
	L	2.17D-06	2.37D-06	2.19D-06	2.15D-06
	Z	1.91D-01	1.92D-01	1.92D-01	1.99D-01
8	P	6.70D-06	8.97D-06	6.67D-06	6.75D-06
	L	1.23D-06	1.51D-06	1.24D-06	1.26D-06
	Z	3.79D-01	4.77D-01	3.83D-01	3.85D-01
9	P	6.39D-06	6.96D-06	6.37D-06	6.61D-06
	L	1.17D-06	1.22D-06	1.19D-06	1.17D-06
	Z	3.72D-01	3.85D-01	3.73D-01	3.81D-01

Table 5.2.4

THE VARIANCE OF THE ESTIMABLES FOR THIS
SELECTION OF OBSERVATIONS

TYPE KEY- D INDICATES AN AZIMUTH
(UNITS FOR VARIANCE ARE ARC SECONDS SQUARED)
S INDICATES A DISTANCE
(UNITS FOR VARIANCE ARE METERS SQUARED)

ESTIMABLE NUMBER/TYPE		VARIANCE ALL *OBS	VARIANCE #26 DELETED	VARIANCE #16 DELETED	VARIANCE #18 DELETED
1	D	5.42D-01	5.42D-01	5.43D-01	5.45D-01
2	D	5.78D-01	5.78D-01	5.78D-01	5.79D-01
3	D	4.94D-01	4.96D-01	4.95D-01	4.94D-01
4	D	2.85D-01	2.85D-01	2.85D-01	2.85D-01
5	D	5.30D-01	5.31D-01	5.33D-01	5.32D-01
6	D	5.61D-01	5.62D-01	5.63D-01	5.61D-01
7	D	5.38D-01	5.38D-01	5.38D-01	6.47D-01
8	D	6.09D-01	6.11D-01	6.37D-01	6.10D-01
9	D	2.44D-01	2.45D-01	2.44D-01	2.44D-01
10	D	4.74D-01	4.75D-01	4.74D-01	4.77D-01
11	D	4.73D-01	4.76D-01	5.02D-01	4.96D-01
12	D	3.77D-01	3.77D-01	4.23D-01	4.45D-01
13	D	6.30D-01	6.32D-01	6.35D-01	6.34D-01
14	D	4.42D-01	4.42D-01	4.75D-01	7.21D-01
15	D	5.17D-01	5.19D-01	5.81D-01	8.73D-01
16	D	5.95D-01	5.97D-01	6.13D-01	1.05D+00
17	D	2.62D-01	2.62D-01	2.68D-01	3.44D-01
18	D	4.38D-01	4.38D-01	4.49D-01	4.98D-01
19	D	5.26D-01	5.44D-01	5.27D-01	5.47D-01
20	D	5.08D-01	7.18D-01	5.10D-01	5.35D-01
21	D	6.30D-01	6.60D-01	6.30D-01	6.70D-01
22	D	3.33D-01	3.38D-01	3.35D-01	3.86D-01
23	D	4.69D-01	4.80D-01	4.76D-01	4.71D-01
24	D	6.25D-01	7.03D-01	6.35D-01	6.29D-01
25	D	5.19D-01	5.21D-01	5.22D-01	5.27D-01
26	D	3.15D-01	3.38D-01	3.15D-01	3.15D-01
27	D	3.45D-01	3.48D-01	3.46D-01	3.46D-01
28	D	4.03D-01	4.37D-01	4.03D-01	4.04D-01
29	D	5.67D-01	6.19D-01	5.67D-01	5.67D-01
30	S	1.06D-03	1.07D-03	1.07D-03	1.07D-03

Table 5.2.5

THE VARIANCE OF THE ESTIMABLES FOR THIS
SELECTION OF OBSERVATIONS
(CONTINUED)

ESTIMABLE NUMBER/TYPE		VARIANCE ALL *OBS	VARIANCE #26 DELETED	VARIANCE #16 DELETED	VARIANCE #18 DELETED
31	S	2.95D-03	2.96D-03	2.98D-03	2.95D-03
32	S	7.88D-03	7.92D-03	7.92D-03	7.89D-03
33	S	4.33D-03	4.33D-03	4.35D-03	4.36D-03
34	S	8.05D-03	8.10D-03	8.10D-03	8.05D-03
35	S	4.77D-03	4.82D-03	4.79D-03	4.78D-03
36	S	6.21D-03	6.25D-03	6.42D-03	6.71D-03
37	S	7.17D-03	7.22D-03	7.63D-03	8.66D-03
38	S	3.55D-03	3.62D-03	3.60D-03	4.56D-03
39	S	5.60D-03	5.66D-03	5.63D-03	6.06D-03
40	S	6.14D-03	6.42D-03	6.17D-03	6.79D-03
41	S	1.94D-03	1.94D-03	1.94D-03	2.06D-03
42	S	2.95D-03	3.09D-03	2.96D-03	3.11D-03
43	S	3.31D-03	3.37D-03	3.34D-03	3.33D-03
44	S	5.54D-03	6.29D-03	5.59D-03	5.57D-03
45	S	6.11D-03	6.72D-03	6.15D-03	6.14D-03
46	S	5.26D-03	7.23D-03	5.29D-03	5.27D-03
47	S	4.12D-03	4.57D-03	4.15D-03	4.13D-03
48	S	1.00D-03	1.00D-03	1.01D-03	1.00D-03

Table 5.2.5 (Continued)

DESCRIPTION OF THE OBSERVATIONS

KEY - D INDICATES A DIRECTION
(UNITS FOR VARIANCE ARE ARC SECONDS SQUARED)

A INDICATES AN AZIMUTH
(UNITS FOR VARIANCE ARE ARC SECONDS SQUARED)

S INDICATES A DISTANCE
(UNITS FOR VARIANCE ARE METERS SQUARED)

OBSERVATION NUMBER	TYPE	A PRIORI VARIANCE	FROM STATION	TO STATION
1	D	0.160	1	2
2	D	0.160	1	3
3	D	0.160	1	4
4	D	0.160	2	1
5	D	0.160	2	3
6	D	0.160	2	4
7	D	0.160	3	1
8	D	0.160	3	2
9	D	0.160	3	4
10	D	0.160	3	5
11	D	0.160	3	6
12	D	0.160	4	1
13	D	0.160	4	2
14	D	0.160	4	3
15	D	0.160	4	5
16	D	0.160	4	6
17	D	0.160	4	7
18	D	0.160	5	3
19	D	0.160	5	4
20	D	0.160	5	6
21	D	0.160	5	7
22	D	0.160	6	3
23	D	0.160	6	4
24	D	0.160	6	5
25	D	0.160	6	7
26	D	0.160	6	8
27	D	0.160	6	9
28	D	0.160	7	4
29	D	0.160	7	5
30	D	0.160	7	6
31	D	0.160	7	8
32	D	0.160	7	9
33	D	0.160	8	6
34	D	0.160	8	7
35	D	0.160	8	9

Table 5.8.1-1

DESCRIPTION OF THE OBSERVATIONS
(CONTINUED)

OBSERVATION NUMBER	TYPE	A PRIORI VARIANCE	FROM STATION	TO STATION
36	D	0.160	9	6
37	D	0.160	9	7
38	D	0.160	9	8
39	A	0.203	1	2
40	A	0.203	1	3
41	A	0.203	1	4
42	A	0.203	2	3
43	A	0.203	2	4
44	A	0.203	3	4
45	A	0.203	3	5
46	A	0.203	3	6
47	A	0.203	4	5
48	A	0.203	4	6
49	A	0.203	4	7
50	A	0.203	5	6
51	A	0.203	5	7
52	A	0.203	6	7
53	A	0.203	6	8
54	A	0.203	6	9
55	A	0.203	7	8
56	A	0.203	7	9
57	A	0.203	8	9
58	S	0.113X10**-2	1	2
59	S	0.116X10**-2	1	3
60	S	0.118X10**-2	1	4
61	S	0.968X10**-3	2	3
62	S	0.205X10**-2	2	4
63	S	0.122X10**-2	3	4
64	S	0.109X10**-2	3	5
65	S	0.112X10**-2	3	6
66	S	0.866X10**-3	4	5
67	S	0.137X10**-2	4	6
68	S	0.117X10**-2	4	7
69	S	0.629X10**-3	5	6
70	S	0.836X10**-3	5	7
71	S	0.115X10**-2	6	7
72	S	0.158X10**-2	6	8
73	S	0.954X10**-3	6	9
74	S	0.937X10**-3	7	8
75	S	0.136X10**-2	7	9
76	S	0.107X10**-2	8	9

Table 5.8.I-I (Continued)

DESCRIPTION OF THE ESTIMABLES IN THIS TEST

KEY - A INDICATES AN AZIMUTH
(UNITS FOR VARIANCE ARE ARC SECONDS SQUARED)

S INDICATES A DISTANCE
(UNITS FOR VARIANCE ARE METERS SQUARED)

G INDICATES AN ANGLE
(UNITS FOR VARIANCE ARE ARC SECONDS SQUARED)

ESTIMABLE NUMBER	TYPE	TARGET VARIANCE	FROM	TO
1	A	0.40	1	2
2	A	0.40	1	3
3	A	0.40	1	4
4	A	0.40	2	3
5	A	0.40	2	4
6	A	0.40	3	4
7	A	0.40	3	5
8	A	0.40	3	6
9	A	0.40	4	5
10	A	0.40	4	6
11	A	0.40	4	7
12	A	0.40	5	6
13	A	0.40	5	7
14	A	0.40	6	7
15	A	0.40	6	8
16	A	0.40	6	9
17	A	0.40	7	8
18	A	0.40	7	9
19	A	0.40	8	9
20	S	0.17X10**-1	1	2
21	S	0.18X10**-1	1	3
22	S	0.18X10**-1	1	4
23	S	0.13X10**-1	2	3
24	S	0.34X10**-1	2	4
25	S	0.19X10**-1	3	4
26	S	0.16X10**-1	3	5
27	S	0.17X10**-1	3	6
28	S	0.12X10**-1	4	5
29	S	0.22X10**-1	4	6
30	S	0.18X10**-1	4	7
31	S	0.65X10**-2	5	6
38	S	0.11X10**-1	5	7
33	S	0.17X10**-1	6	7
34	S	0.26X10**-1	6	8
35	S	0.13X10**-1	6	9
36	S	0.13X10**-1	7	8
37	S	0.22X10**-1	7	9
38	S	0.16X10**-1	8	9

Table 5.8.1-2

ABSTRACT OF PREDICTED OBSERVATION
CORRELATION

EXPLANATION OF TABLE- COLUMN A IS THE OBSERVATION BEING CONSIDERED

COLUMN B ARE THE OTHER OBSERVATIONS WITH WHICH THE COLUMN A OBSERVATION IS CORRELATED WITH COEFFICIENTS OF BETWEEN 1. AND .80

COLUMN C ARE THE OTHER OBSERVATIONS WITH WHICH THE COLUMN A OBSERVATION IS CORRELATED WITH COEFFICIENTS OF BETWEEN .79 AND .55

COLUMN A	COLUMN B	COLUMN C
1	NONE	2, 39
2	NONE	1, 3, 40
3	NONE	2, 41
4	NONE	6, 5, 39
5	NONE	6, 42, 4
6	NONE	4, 5
7	NONE	8, 9, 40
8	NONE	42, 7
9	NONE	7, 10, 11, 44
10	NONE	11, 45, 9
11	NONE	10, 46, 9
12	NONE	13, 14, 41
13	14	12, 16, 15
14	13	16, 12, 15, 44
15	16	17, 13, 14, 47

Table 5.8.1-3

ABSTRACT OF PREDICTED OBSERVATION
CORRELATION (CONTINUED)

COLUMN A	COLUMN B	COLUMN C
*****	************************	************************
16	15	13, 14, 17, 48
17	NONE	15, 16, 49
18	NONE	45, 19, 20
19	NONE	18, 47
20	NONE	50, 18, 24
21	NONE	51, 29
22	NONE	46, 23, 24
23	NONE	24, 22, 25, 26, 48
24	NONE	23, 25, 50, 20, 22, 26
25	NONE	24, 26, 23, 52
26	NONE	25, 27, 23, 24, 53
27	NONE	26, 54
28	NONE	29, 49
29	NONE	51, 21, 28, 30, 32
30	NONE	32, 29, 52
31	NONE	55, 32
32	NONE	30, 29, 31, 56
33	NONE	34, 35, 53
34	NONE	33, 55, 35
35	NONE	33, 34, 57

Table 5.8.1-3 (Continued)

ABSTRACT OF PREDICTED OBSERVATION
CORRELATION (CONTINUED)

COLUMN A	COLUMN B	COLUMN C
*****	*************************	*************************
36	NONE	54, 37
37	NONE	38, 36, 56
38	NONE	37, 57
39	NONE	1, 4
40	NONE	2,7
41	NONE	3, 12
42	NONE	5, 8
43	NONE	NONE
44	NONE	9, 14, 64, 65
45	NONE	10, 18
46	NONE	11, 22
47	NONE	48, 15, 19, 63
48	NONE	47, 16, 23, 50
49	NONE	17, 28
50	NONE	20, 24, 48, 72
51	NONE	21, 29
52	NONE	25, 30, 68
53	NONE	26, 33
54	NONE	27, 36
55	NONE	31, 34
56	NONE	32, 37, 70
57	NONE	35, 38

Table 5.8.1-3 (Continued)

ABSTRACT OF PREDICTED OBSERVATION
CORRELATION (CONTINUED)

COLUMN A	COLUMN B	COLUMN C
*****	*************************	*************************
58	NONE	59
59	NONE	58
60	NONE	62
61	NONE	NONE
62	NONE	60, 63
63	NONE	47, 62
64	NONE	44, 65
65	NONE	44, 64
66	NONE	NONE
67	NONE	NONE
68	NONE	52
69	NONE	NONE.
70	NONE	56, 68, 71
71	NONE	70, 72
72	NONE	50, 71, 73, 76
73	NONE	72
74	NONE	NONE
75	NONE	76
76	NONE	72, 75

Table 5.8.1-3 (Continued)

SELECTION OF OBSERVATIONS FOR THE FORMATION OF THE V/C MATRIX FOR POSITIONAL PARAMETERS

EXPLANATION OF THE ENTRIES IN THE 'SELECTION' COLUMN

I - INDICATES A SELECTION (INITIAL FORMATION)

A - INDICATES AN ADDITION. THE NUMBER FOLLOWING IS THE ITERATION IN WHICH THIS OBSERVATION WAS ADDED

KEPT - INDICATES THAT THIS OBSERVATION WAS INCLUDED TO INSURE THAT AT LEAST TWO DIRECTIONS WERE OBSERVED AT EACH STATION

.J - WHERE J IS A NUMBER FROM 1 TO 9 INDICATES THAT THIS OBSERVATION WAS CORRELATED TO ANOTHER SELECTED OBSERVATION WITH A COEFFICIENT OF .J

OBSERVATION NUMBER	SELECTION	OBSERVATION NUMBER	SELECTION
1	I	21	I
2	I	22	I
3	I	23 .6	
4	I	24 .6	
5	I	25	I
6 .6	A1	26 .7	A1
7	I	27	I
8 .6	A1	28	I
9 .6		29 .6	
10	I	30 .6	
11	I	31	I
12	I	32	I
13 .7		33 .6	A2
14 .6		34 .6	A2
15	I	35	I
16	I	36	I
17	I	37 .6	A1
18	I	38	I
19 .6		39 .6	
20	I	40 .6	

Table 5.8.1-4

SELECTION OF OBSERVATIONS FOR THE FORMATION OF THE V/C
MATRIX FOR POSITIONAL PARAMETERS
(CONTINUED)

OBSERVATION NUMBER	SELECTION	OBSERVATION NUMBER	SELECTION
41	.6	61	I
42	.7	62	.6
43	I	63	I
44	.6	64	I
45	.7	65	.6
46	.7	66	I
47	.6	67	I
48	.6	68	I
49	.6	69	I
50	.7	70	.6
51	.7	71	I
52	.6	72	.6
53	.6	73	I
54	.7	74	I
55	.7	75	I
56	.6	76	.6
57	.6		
58	.6		
59	I		
60	I		

Table 5.8.1-4 (Continued)

STATION VARIANCES
FOR THIS NET

KEY TO PARAMETER CODE- P INDICATES THE VARIANCE IN LATITUDE
L INDICATES THE VARIANCE IN LONGITUDE
Z INDICATES THE VARIANCE IN STATION
UNKNOWN ('Z' CORRECTION)

NOTE THAT ALL PARAMETER VARIANCES ARE IN UNITS OF ARC SECONDS SQUARED AND THE HEADING 'CODE' INDICATES PARAMETER CODE

STATION NUMBER/CODE		VARIANCE INITIAL	1 VARIANCE ITERATION	2 VARIANCE ITERATION (FINAL)	VARIANCE MINIMUM CONSTRAINT
1	P	6.07D-07	5.84D-07	5.85D-07	1.00D-10*
	L	8.77D-06	7.11D-06	7.73D-06	1.00D-10*
	Z	3.38D-01	3.07D-01	3.07D-01	3.08D-01
2	P	1.25D-06	1.24D-06	1.23D-06	1.85D-06
	L	6.31D-06	6.32D-06	6.89D-06	2.23D-06
	Z	3.48D-01	2.69D-01	2.69D-01	2.70D-01
3	P	8.28D-07	6.62D-07	6.66D-07	1.24D-06
	L	2.22D-06	1.69D-06	2.01D-06	2.89D-06
	Z	4.00D-01	2.77D-01	2.77D-01	2.79D-01
4	P	8.61D-07	7.24D-07	7.22D-07	7.63D-07
	L	1.34D-06	1.15D-06	1.39D-06	3.59D-06
	Z	4.23D-01	3.41D-01	3.40D-01	3.42D-01
5	P	1.54D-07	1.57D-07	1.56D-07	8.39D-07
	L	3.73D-07	3.68D-07	3.50D-07	8.82D-06
	Z	4.49D-01	3.55D-01	3.55D-01	3.56D-01
6	P	6.50D-07	5.34D-07	5.31D-07	1.73D-06
	L	6.39D-07	5.11D-07	3.96D-07	1.07D-05
	Z	4.67D-01	3.65D-01	3.65D-01	3.67D-01
7	P	1.16D-06	9.51D-07	9.46D-07	1.29D-06
	L	1.91D-06	1.48D-06	1.20D-06	1.42D-05
	Z	5.27D-01	4.22D-01	4.16D-01	4.18D-01
8	P	1.35D-06	1.05D-06	9.44D-07	1.52D-06
	L	1.16D-05	8.23D-06	7.40D-06	2.87D-05
	Z	7.59D-01	6.40D-01	4.55D-01	4.57D-01
9	P	1.32D-06	1.14D-06	1.08D-06	2.35D-06
	L	6.89D-06	5.28D-06	4.74D-06	2.34D-05
	Z	6.00D-01	4.54D-01	4.48D-01	4.50D-01

(NOTE THAT "*" INDICATES A WEIGHT CONSTRAINED PARAMETER)

Table 5.8.1-5

THE VARIANCE OF THE ESTIMABLES FOR THIS
SELECTION OF OBSERVATIONS

TYPE KEY- A INDICATES AN AZIMUTH
(UNITS FOR VARIANCE ARE ARC SECONDS SQUARED)
S INDICATES A DISTANCE
(UNITS FOR VARIANCE ARE METERS SQUARED)

ESTIMABLE NUMBER/TYPE		VARIANCE INITIAL	VARIANCE 1 ITERATION	VARIANCE 2 ITERATION (FINAL)	VARIANCE MINIMUM CONSTRAINT
1	A	2.71D-01	2.65D-01	2.65D-01	2.65D-01
2	A	3.90D-01	3.28D-01	3.27D-01	3.27D-01
3	A	3.55D-01	2.89D-01	2.89D-01	2.89D-01
4	A	4.13D-01	2.75D-01	2.75D-01	2.75D-01
5	A	2.03D-01	2.03D-01	2.03D-01	2.03D-01
6	A	3.44D-01	2.60D-01	2.60D-01	2.60D-01
7	A	3.73D-01	2.76D-01	2.76D-01	2.76D-01
8	A	4.02D-01	3.02D-01	3.01D-01	3.01D-01
9	A	4.91D-01*	4.05D-01	4.04D-01	4.04D-01
10	A	4.18D-01	3.30D-01	3.30D-01	3.30D-01
11	A	4.65D-01*	3.67D-01	3.67D-01	3.67D-01
12	A	5.66D-01*	4.70D-01*	4.48D-01	4.48D-01
13	A	4.75D-01*	3.84D-01	3.84D-01	3.84D-01
14	A	4.60D-01*	3.66D-01	3.66D-01	3.66D-01
15	A	5.15D-01*	3.84D-01	3.84D-01	3.84D-01
16	A	5.33D-01*	4.28D-01	4.28D-01	4.28D-01
17	A	6.85D-01*	4.92D-01*	4.36D-01	4.36D-01
18	A	5.11D-01*	4.08D-01	4.01D-01	4.01D-01
19	A	5.97D-01*	4.78D-01*	4.50D-01	4.50D-01
20	S	2.56D-03	1.27D-03	1.27D-03	1.27D-03
21	S	5.68D-04	5.64D-04	5.63D-04	5.63D-04
22	S	7.41D-04	6.89D-04	6.89D-04	6.89D-04
23	S	6.43D-04	6.33D-04	6.33D-04	6.33D-04
24	S	1.51D-03	1.09D-03	1.09D-03	1.09D-03
25	S	4.00D-04	3.95D-04	3.92D-04	3.92D-04
26	S	6.73D-04	6.69D-04	6.69D-04	6.69D-04
27	S	9.01D-04	8.95D-04	8.94D-04	8.94D-04
28	S	3.57D-04	3.48D-04	3.47D-04	3.47D-04
29	S	4.05D-04	3.92D-04	3.81D-04	3.81D-04
30	S	6.28D-04	6.14D-04	6.13D-04	6.13D-04

(NOTE THAT "*" INDICATES ESTIMABLE FAILS USER REQUIREMENT)

Table 5.8.1-5 (Continued)

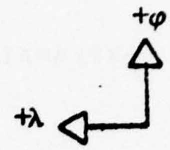
THE VARIANCE OF THE ESTIMABLES FOR THIS
SELECTION OF OBSERVATIONS
(CONTINUED)

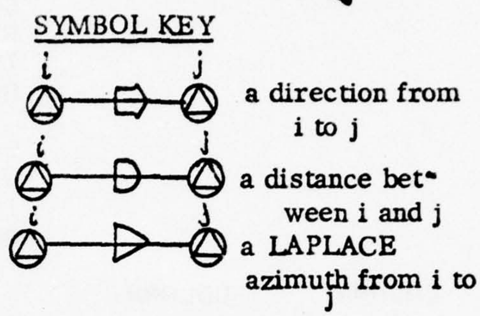
ESTIMABLE NUMBER/TYPE		VARIANCE INITIAL	1 VARIANCE ITERATION	2 VARIANCE ITERATION (FINAL)	VARIANCE MINIMUM CONSTRAINT
31	S	2.81D-04	2.75D-04	2.71D-04	2.71D-04
32	S	5.70D-04	5.21D-04	5.18D-04	5.18D-04
33	S	4.16D-04	3.47D-04	3.33D-04	3.33D-04
34	S	1.37D-03	8.76D-04	7.08D-04	7.08D-04
35	S	6.69D-04	6.63D-04	5.90D-04	5.90D-04
36	S	7.45D-04	5.71D-04	5.13D-04	5.13D-04
37	S	5.12D-04	4.36D-04	4.06D-04	4.06D-04
38	S	1.60D-03	6.07D-04	5.26D-04	5.26D-04

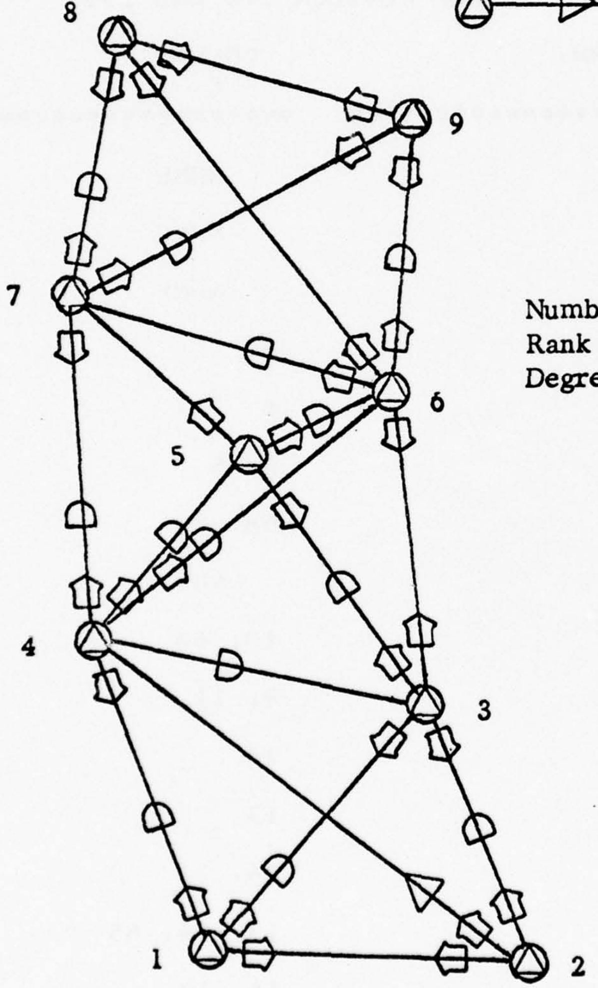
(NOTE THAT "*" INDICATES ESTIMABLE FAILS USER REQUIREMENT)

Table 5.8.1-5 (Continued)

Observations required in the design of example 5-1
NETWORK T3







Number of Observations - 44
Rank of free normals - 25
Degrees of freedom - 19

Stations (△) joined by solid lines indicate that they are intervisible

Figure 5.8.1-1

ABSTRACT OF PREDICTED OBSERVATION
CORRELATION

EXPLANATION OF TABLE- COLUMN A IS THE OBSERVATION BEING CONSIDERED

COLUMN B ARE THE OTHER OBSERVATIONS WITH WHICH THE COLUMN A OBSERVATION IS CORRELATED WITH COEFFICIENTS OF BETWEEN 1. AND .80

COLUMN C ARE THE OTHER OBSERVATIONS WITH WHICH THE COLUMN A OBSERVATION IS CORRELATED WITH COEFFICIENTS OF BETWEEN .79 AND .55

COLUMN A	COLUMN B	COLUMN C
1	NONE	NONE
2	NONE	NONE
3	NONE	NONE
4	NONE	6
5	NONE	6
6	NONE	4, 5
7	NONE	58
8	NONE	NONE
9	NONE	10, 60
10	NONE	9, 11
11	NONE	10
12	NONE	13
13	NONE	14, 12
14	NONE	13, 64, 65
15	NONE	16, 19
16	NONE	15, 17

Table 5.8.1-o

ABSTRACT OF PREDICTED OBSERVATION
CORRELATION (CONTINUED)

COLUMN A	COLUMN B	COLUMN C
*****	**************************	**************************
17	NONE	16
18	NONE	NONE
19	NONE	15
20	NONE	24
21	NONE	NONE
22	NONE	23
23	NONE	22
24	NONE	20
25	NONE	NONE
26	NONE	NONE
27	NONE	NONE
28	NONE	29
29	NONE	30, 28
30	NONE	29, 32
31	NONE	NONE
32	NONE	30
33	NONE	35
34	NONE	NONE
35	NONE	33

Table 5.8.1-6(Continued)

ABSTRACT OF PREDICTED OBSERVATION
CORRELATION (CONTINUED)

COLUMN A	COLUMN B	COLUMN C
*****	************************	************************
36	NONE	NONE
37	NONE	NONE
38	NONE	NONE
39	40, 43	41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57
40	39, 41	43, 44, 45, 46, 47, 48, 49, 52, 42, 50, 51, 53, 54, 55, 56, 57
41	40, 43, 45, 46	39, 44, 47, 48, 49, 51, 52, 53, 56, 42, 50, 54, 55, 57
42	43	39, 40, 41, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56
43	39, 41, 42, 44, 45, 46	40, 47, 48, 49, 51, 52, 53, 56, 50, 54, 55, 57
44	43, 45	39, 40, 41, 46, 47, 48, 49, 51, 52, 53, 42, 50, 54, 56, 57
45	41, 43, 44, 46, 48, 49, 52 53	39, 40, 47, 50, 51, 54, 55, 56, 57, 42
46	41, 43, 45, 48, 51, 52, 53 56	39, 40, 44, 47, 50, 54, 55, 57, 42
47	48, 49	40, 41, 43, 44, 45, 46, 52, 53, 55, 56, 39, 42, 50, 51, 54, 57
48	45, 46, 47, 49, 50, 51, 52 53, 56	40, 41, 43, 44, 54, 55, 57, 39, 42

Table 5.8.1-6 (Continued)

ABSTRACT OF PREDICTED OBSERVATION
CORRELATION (CONTINUED)

COLUMN A	COLUMN B	COLUMN C
*****	*************************	*************************
49	45, 46, 47, 48, 51, 52, 53 54, 56	40, 41, 43, 44, 50, 55, 57, 39, 42
50	48, 52	55, 46, 49, 51, 53, 54, 45, 56, 57, 39, 40, 41, 43, 44, 47
51	46, 48, 49, 52, 53, 54, 56	41, 43, 44, 45, 50, 55, 57, 39, 40, 42, 47
52	45, 46, 48, 49, 50, 51, 53 54, 55, 56, 57	40, 41, 43, 44, 47, 39, 42
53	45, 46, 48, 49, 51, 52, 54 55, 56, 57	41, 43, 44, 47, 50, 39, 40, 42
54	49, 51, 52, 53, 55, 56, 57	45, 46, 48, 50, 39, 40, 41, 42, 43, 44, 47
55	52, 53, 54, 56, 57	45, 46, 47, 48, 49, 50, 51, 39, 40, 41, 42, 43, 44
56	46, 48, 49, 51, 52, 53, 54 55, 57	41, 43, 45, 47, 50, 39, 40, 42, 44
57	52, 53, 54, 55, 56	45, 46, 48, 49, 50, 51 39, 40, 41, 43, 44, 47
58	NONE	59, 62
59	NONE	58
60	NONE	9
61	NONE	62
62	NONE	58, 61, 63
63	NONE	62
64	65	14

Table 5.8.1-o (Continued)

ABSTRACT OF PREDICTED OBSERVATION
CORRELATION (CONTINUED)

COLUMN A	COLUMN B	COLUMN C
*****	**************************	**************************
65	64	14
66	NONE	67
67	NONE	66
68	NONE	NONE
69	NONE	NONE
70	NONE	71
71	NONE	70, 72
72	NONE	71, 73, 76
73	NONE	72
74	NONE	NONE
75	NONE	NONE
76	NONE	72

Table 5.8.1-o (Continued)

SELECTION OF OBSERVATIONS FOR THE FORMATION OF THE V/C MATRIX FOR POSITIONAL PARAMETERS

EXPLANATION OF THE ENTRIES IN THE 'SELECTION' COLUMN

- I - INDICATES A SELECTION (INITIAL FORMATION)
- A - INDICATES AN ADDITION. THE NUMBER FOLLOWING IS THE ITERATION IN WHICH THIS OBSERVATION WAS ADDED
- KEPT - INDICATES THAT THIS OBSERVATION WAS INCLUDED TO INSURE THAT AT LEAST TWO DIRECTIONS WERE OBSERVED AT EACH STATION
- .J - WHERE J IS A NUMBER FROM 1 TO 9 INDICATES THAT THIS OBSERVATION WAS CORRELATED TO ANOTHER SELECTED OBSERVATION WITH A COEFFICIENT OF .J

OBSERVATION NUMBER	SELECTION	OBSERVATION NUMBER	SELECTION
1	I	21	I
2	I	22	I
3	I	23 .6	
4	I	24 .6	
5	I	25	I
6 .6		26	I
7	I	27	I
8	I	28	I
9	I	29 .6	
10 .6		30	I
11	I	31	I
12	I	32 .6	A2
13 .6		33	I
14 .6		34	I
15	I	35 .6	A2
16 .6		36	I
17	I	37	I
18	I	38	I
19 .6		39 .6	
20	I	40 .7	

Table 5.8.1-7

SELECTION OF OBSERVATIONS FOR THE FORMATION OF THE V/C MATRIX FOR POSITIONAL PARAMETERS (CONTINUED)

OBSERVATION NUMBER		SELECTION	OBSERVATION NUMBER		SELECTION
41	.7		61	.6	A1
42	.6		62		I
43	.7		63	.6	
44	.7		64	.6	
45	.8		65		I
46	.8		66		I
47	.8		67	.6	
48		I	68		I
49	.8		69		I
50	.8		70		I
51	.8		71	.6	
52	.8		72		I
53	.8		73	.6	
54	.7		74		I
55	.7		75		I
56	.8		76	.6	A1
57	.7				
58	.6				
59		I			
60	.6				

Table 5.8.1-7 (Continued)

STATION VARIANCES
FOR THIS NET

KEY TO PARAMETER CODE- P INDICATES THE VARIANCE IN LATITUDE
L INDICATES THE VARIANCE IN LONGITUDE
Z INDICATES THE VARIANCE IN STATION
UNKNOWN ('Z' CORRECTION)

NOTE THAT ALL PARAMETER VARIANCES ARE IN UNITS OF ARC SECONDS SQUARED AND THE HEADING 'CODE' INDICATES PARAMETER CODE

STATION NUMBER/CODE		VARIANCE INITIAL	VARIANCE 1 ITERATION	VARIANCE 2 ITERATION (FINAL)	VARIANCE MINIMUM CONSTRAINT
1	P	8.76D-07	8.58D-07	8.85D-07	1.00D-10*
	L	7.34D-06	7.60D-06	7.68D-06	1.00D-10*
	Z	4.15D-01	4.07D-01	4.07D-01	4.09D-01
2	P	2.83D-06	2.20D-06	2.26D-06	3.07D-06
	L	8.08D-06	8.40D-06	8.47D-06	2.08D-06
	Z	4.98D-01	4.80D-01	4.80D-01	4.82D-01
3	P	1.09D-06	9.52D-07	9.91D-07	1.68D-06
	L	1.95D-06	2.13D-06	2.16D-06	3.00D-06
	Z	3.27D-01	3.27D-01	3.27D-01	3.29D-01
4	P	5.57D-07	6.08D-07	6.09D-07	1.23D-06
	L	8.65D-07	9.71D-07	9.94D-07	4.75D-06
	Z	2.98D-01	2.96D-01	2.96D-01	2.98D-01
5	P	1.43D-07	1.57D-07	1.57D-07	1.23D-06
	L	3.19D-07	3.17D-07	3.16D-07	8.62D-06
	Z	3.25D-01	3.25D-01	3.25D-01	3.26D-01
6	P	4.05D-07	3.80D-07	3.76D-07	1.73D-06
	L	4.09D-07	3.65D-07	3.56D-07	9.97D-06
	Z	3.29D-01	3.27D-01	3.25D-01	3.26D-01
7	P	8.23D-07	9.02D-07	8.74D-07	1.82D-06
	L	1.29D-06	1.14D-06	1.10D-06	1.38D-05
	Z	3.25D-01	3.19D-01	3.14D-01	3.15D-01
8	P	9.12D-07	9.66D-07	9.00D-07	2.01D-06
	L	6.73D-06	6.08D-06	6.00D-06	2.56D-05
	Z	4.28D-01	4.14D-01	3.96D-01	3.97D-01
9	P	3.63D-06	2.58D-06	1.22D-06	2.85D-06
	L	4.72D-06	3.97D-06	3.81D-06	2.10D-05
	Z	6.28D-01	5.17D-01	4.12D-01	4.13D-01

(NOTE THAT "*" INDICATES A WEIGHT CONSTRAINED PARAMETER)

Table 5.8.1-8

THE VARIANCE OF THE ESTIMABLES FOR THIS
SELECTION OF OBSERVATIONS

TYPE KEY- A INDICATES AN AZIMUTH
(UNITS FOR VARIANCE ARE ARC SECONDS SQUARED)
S INDICATES A DISTANCE
(UNITS FOR VARIANCE ARE METERS SQUARED)

ESTIMABLE NUMBER/TYPE		VARIANCE INITIAL	VARIANCE 1 ITERATION	VARIANCE 2 ITERATION (FINAL)	VARIANCE MINIMUM CONSTRAINT
1	A	4.92D-01*	4.47D-01	4.47D-01	4.47D-01
2	A	3.96D-01	3.95D-01	3.95D-01	3.95D-01
3	A	3.69D-01	3.61D-01	3.61D-01	3.61D-01
4	A	4.29D-01	4.26D-01	4.26D-01	4.26D-01
5	A	3.53D-01	3.40D-01	3.40D-01	3.40D-01
6	A	3.27D-01	3.25D-01	3.25D-01	3.25D-01
7	A	3.55D-01	3.55D-01	3.54D-01	3.54D-01
8	A	2.94D-01	2.94D-01	2.94D-01	2.94D-01
9	A	2.62D-01	2.62D-01	2.61D-01	2.61D-01
10	A	2.03D-01	2.03D-01	2.03D-01	2.02D-01
11	A	2.80D-01	2.77D-01	2.76D-01	2.76D-01
12	A	3.42D-01	3.42D-01	3.41D-01	3.41D-01
13	A	3.57D-01	3.56D-01	3.55D-01	3.55D-01
14	A	2.95D-01	2.94D-01	2.94D-01	2.94D-01
15	A	3.45D-01	3.36D-01	3.35D-01	3.35D-01
16	A	4.90D-01*	4.23D-01	3.94D-01	3.94D-01
17	A	3.88D-01	3.68D-01	3.64D-01	3.64D-01
18	A	6.53D-01*	4.97D-01*	3.56D-01	3.56D-01
19	A	7.36D-01*	6.19D-01*	4.27D-01	4.27D-01
20	S	1.28D-03	1.15D-03	1.15D-03	1.15D-03
21	S	6.69D-04	5.79D-04	5.78D-04	5.78D-04
22	S	1.28D-03	1.27D-03	1.27D-03	1.27D-03
23	S	1.27D-03	5.49D-04	5.49D-04	5.49D-04
24	S	1.20D-03	9.22D-04	9.20D-04	9.20D-04
25	S	5.82D-03	5.75D-04	5.72D-04	5.72D-04
26	S	6.99D-04	6.80D-04	6.80D-04	6.80D-04
27	S	9.71D-04	9.52D-04	9.51D-04	9.51D-04
28	S	4.02D-04	4.02D-04	4.01D-04	4.01D-04
29	S	5.10D-04	5.06D-04	5.05D-04	5.05D-04
30	S	6.08D-04	6.07D-04	6.06D-04	6.06D-04

(NOTE THAT "*" INDICATES ESTIMABLE FAILS USER REQUIREMENT)

Table 5.8.1-8 (Continued)

THE VARIANCE OF THE ESTIMABLES FOR THIS
SELECTION OF OBSERVATIONS
(CONTINUED)

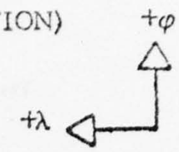
ESTIMABLE NUMBER/TYPE		VARIANCE INITIAL	VARIANCE 1 ITERATION	VARIANCE 2 ITERATION (FINAL)	VARIANCE MINIMUM CONSTRAINT
31	S	3.19D-04	3.13D-04	3.13D-04	3.13D-04
32	S	3.54D-04	3.47D-04	3.42D-04	3.42D-04
33	S	4.13D-04	3.83D-04	3.76D-04	3.76D-04
34	S	6.86D-04	5.75D-04	5.65D-04	5.65D-04
35	S	3.23D-03	2.23D-03	9.62D-04	9.62D-04
36	S	5.04D-04	5.03D-04	4.86D-04	4.86D-04
37	S	6.01D-04	5.93D-04	4.67D-04	4.67D-04
38	S	2.10D-03	7.11D-04	4.80D-04	4.80D-04

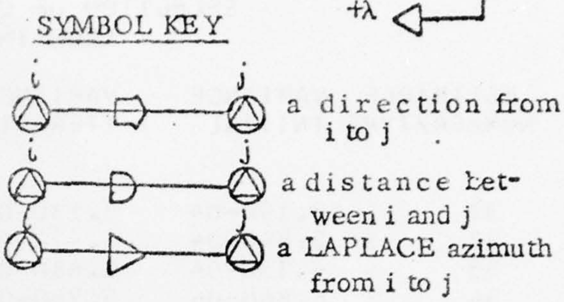
(NOTE THAT "*" INDICATES ESTIMABLE FAILS USER REQUIREMENT)

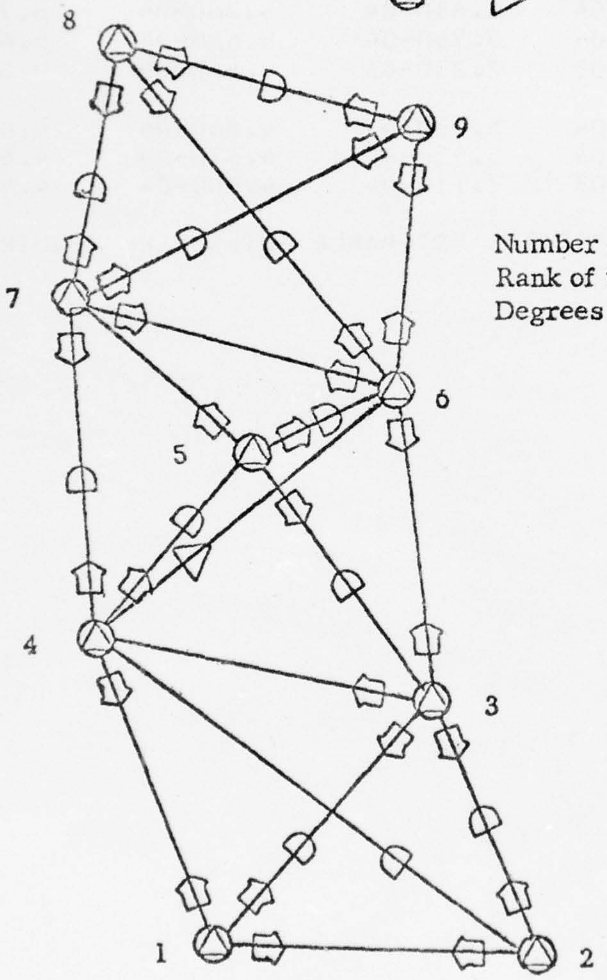
Table 5.8.1-8 (Continued)

Observations Required in the design of example 5-1 (ITERATION)

NETWORK T3







Number of Observations - 42
Rank of the Free Normals - 25
Degrees of freedom - 17

Stations (△) joined by solid lines indicate that they are intervisible

Figure 5.8.1-2

DESCRIPTION OF THE OBSERVATIONS

KEY - D INDICATES A DIRECTION
(UNITS FOR VARIANCE ARE ARC SECONDS SQUARED)

A INDICATES AN AZIMUTH
(UNITS FOR VARIANCE ARE ARC SECONDS SQUARED)

S INDICATES A DISTANCE
(UNITS FOR VARIANCE ARE METERS SQUARED)

OBSERVATION NUMBER	TYPE	A PRIORI VARIANCE	FROM STATION	TO STATION
1	D	0.320	1	2
2	D	0.320	1	5
3	D	0.320	1	6
4	D	0.320	2	1
5	D	0.320	2	3
6	D	0.320	2	4
7	D	0.320	2	5
8	D	0.320	2	6
9	D	0.320	3	2
10	D	0.320	3	4
11	D	0.320	3	6
12	D	0.320	2	4
13	D	0.320	4	3
14	D	0.320	4	6
15	D	0.320	5	1
16	D	0.320	5	2
17	D	0.320	5	6
18	D	0.320	6	1
19	D	0.320	6	2
20	D	0.320	6	3
21	D	0.320	6	4
22	D	0.320	6	5
23	S	0.375X10**-2	1	2
24	S	0.340X10**-2	1	5
25	S	0.482X10**-2	1	6
26	S	0.718X10**-3	2	3
27	S	0.616X10**-3	2	4
28	S	0.275X10**-2	2	5
29	S	0.127X10**-2	2	6
30	S	0.549X10**-3	3	4
31	S	0.800X10**-3	3	6
32	S	0.790X10**-3	4	6
33	S	0.182X10**-2	5	6

Table 5.8.2-1

DESCRIPTION OF THE ESTIMABLES IN THIS TEST

KEY - A INDICATES AN AZIMUTH
(UNITS FOR VARIANCE ARE ARC SECONDS SQUARED)

S INDICATES A DISTANCE
(UNITS FOR VARIANCE ARE METERS SQUARED)

G INDICATES AN ANGLE
(UNITS FOR VARIANCE ARE ARC SECONDS SQUARED)

ESTIMABLE NUMBER	TYPE	TARGET VARIANCE	AT STATION	FROM STATION	TO STATION
1	G	0.89	1	2	5
2	G	0.89	1	2	6
3	G	0.89	2	1	3
4	G	0.89	2	1	4
5	G	0.89	2	1	5
6	G	0.89	2	1	6
7	G	0.89	3	2	4
8	G	0.89	3	2	6
9	G	0.89	4	2	3
10	G	0.89	4	2	6
11	G	0.89	5	1	2
12	G	0.89	5	1	6
13	G	0.89	6	1	2
14	G	0.89	6	1	3
15	G	0.89	6	1	4
16	G	0.89	6	1	5

Table 5.8.2-2

DESCRIPTION OF THE ESTIMABLES IN THIS TEST
(CONTINUED)

ESTIMABLE NUMBER	TYPE	TARGET VARIANCE	FROM	TO
17	S	0.88X10**-2	1	2
18	S	0.79X10**-2	1	5
19	S	0.12X10**-1	1	6
20	S	0.17X10**-2	2	3
21	S	0.14X10**-2	2	4
22	S	0.64X10**-2	2	5
23	S	0.30X10**-2	2	6
24	S	0.13X10**-2	3	4
25	S	0.19X10**-2	3	6
26	S	0.18X10**-2	4	6
27	S	0.42X10**-2	5	6

Table 5.8.2-2 (Continued)

ABSTRACT OF PREDICTED OBSERVATION
CORRELATION

EXPLANATION OF TABLE- COLUMN A IS THE OBSERVATION BEING CONSIDERED

COLUMN B ARE THE OTHER OBSERVATIONS WITH WHICH THE COLUMN A OBSERVATION IS CORRELATED WITH COEFFICIENTS OF BETWEEN 1. AND .80

COLUMN C ARE THE OTHER OBSERVATIONS WITH WHICH THE COLUMN A OBSERVATION IS CORRELATED WITH COEFFICIENTS OF BETWEEN .79 AND .55

COLUMN A	COLUMN B	COLUMN C
1	2, 3	11, 20
2	1, 3	8, 14, 19, 21
3	1, 2	8, 19
4	6, 8, 5, 7	11, 20
5	6, 7, 8, 4	9, 12, 14, 19, 21
6	4, 5, 7, 8, 12	9, 10, 11, 13, 14, 19, 20, 21
7	5, 6, 8, 4	11, 16, 17, 20, 22
8	4, 5, 6, 7, 14, 19, 21	2, 3, 9, 11, 12, 15, 18, 20
9	11, 10	5, 6, 8, 12, 14, 19, 21
10	9, 11, 13	6, 12
11	9, 10, 20	1, 4, 6, 7, 8, 12, 14, 16, 19, 21, 23
12	13, 14, 6	5, 8, 9, 10, 11, 19, 20, 21
13	12, 10, 14	6
14	12, 8, 13, 19, 21	2, 5, 6, 9, 11, 15, 20

Table 5.8.2-3

ABSTRACT OF PREDICTED OBSERVATION
CORRELATION (CONTINUED)

COLUMN A	COLUMN B	COLUMN C
*****	**************************	**************************
15	NONE	8, 14, 16, 17, 19, 21
16	17	7, 11, 15, 20, 22
17	16	7, 15, 22, 23
18	19, 20, 21	8, 22
19	18, 20, 21, 8, 14	2, 3, 5, 6, 9, 11, 12, 15, 22
20	18, 19, 21, 11	1, 4, 6, 8, 12, 14, 16 22, 23
21	18, 19, 20, 8, 14	2, 5, 6, 9, 11, 12, 15
22	NONE	7, 16, 19, 20, 23
23	25	11, 17, 20, 22, 30
24	31	NONE
25	23, 26, 30	33
26	25, 27, 28, 30	29
27	33, 26	29
28	29, 31, 26, 32	30, 33
29	28, 33, 31	26, 27, 32
30	33, 25, 26	23, 28
31	28, 33, 24, 32	NONE
32	28, 31	29
33	26, 29, 30, 31, 27	25, 28

Table 5.8.2-3 (Continued)

SELECTION OF OBSERVATIONS FOR THE FORMATION OF THE V/C MATRIX FOR POSITIONAL PARAMETERS

EXPLANATION OF THE ENTRIES IN THE 'SELECTION' COLUMN
I - INDICATES A SELECTION (INITIAL FORMATION)
A - INDICATES AN ADDITION. THE NUMBER FOLLOWING IS THE ITERATION IN WHICH THIS OBSERVATION WAS ADDED
KEPT - INDICATES THAT THIS OBSERVATION WAS INCLUDED TO INSURE THAT AT LEAST TWO DIRECTIONS WERE OBSERVED AT EACH STATION
.J - WHERE J IS A NUMBER FROM 1 TO 9 INDICATES THAT THIS OBSERVATION WAS CORRELATED TO ANOTHER SELECTED OBSERVATION WITH A COEFFICIENT OF .J

OBSERVATION NUMBER		SELECTION	OBSERVATION NUMBER		SELECTION
1		I	21	.8	A1
2	.8	A2	22		I
3	KEPT		23		I
4		I	24		I
5	.7	A1	25	.8	
6	.8	A1	26		I
7	KEPT		27	.7	
8	.8	A2	28	.8	A3
9		I	29		I
10	KEPT		30		I
11	.8		31	.7	
12		I	32		I
13	.8		33	1.	
14	KEPT				
15		I			
16		I			
17	.8	A1			
18		I			
19	.8	A1			
20	.8				

Table 5.8.2-4

STATION VARIANCES
FOR THIS NET

KEY TO PARAMETER CODE- P INDICATES THE VARIANCE IN LATITUDE
L INDICATES THE VARIANCE IN LONGITUDE
Z INDICATES THE VARIANCE IN STATION
UNKNOWN ('Z' CORRECTION)

NOTE THAT ALL PARAMETER VARIANCES ARE IN UNITS OF ARC SECONDS SQUARED AND THE HEADING 'CODE' INDICATES PARAMETER CODE

STATION NUMBER/CODE		VARIANCE INITIAL	VARIANCE ITERATION 1	VARIANCE ITERATION 2	VARIANCE ITERATION 3 (FINAL)
1	P	2.17D-05	3.07D-06	2.77D-06	2.49D-06
	L	1.98D-05	4.37D-06	4.33D-06	4.06D-06
	Z	5.22D-01	1.98D-01	1.40D-01	1.38D-01
2	P	1.94D-06	2.57D-07	2.51D-07	2.25D-07
	L	2.64D-06	1.62D-07	1.60D-07	1.60D-07
	Z	5.93D-01	8.21D-02	6.30D-02	6.31D-02
3	P	1.16D-06	2.95D-07	2.86D-07	2.83D-07
	L	4.41D-07	3.57D-07	3.55D-07	3.52D-07
	Z	3.13D+00	4.81D-01	4.80D-01	4.80D-01
4	P	7.83D-07	2.21D-07	2.22D-07	2.29D-07
	L	7.68D-07	1.37D-07	1.37D-07	1.33D-07
	Z	2.45D+00	2.34D-01	2.16D-01	2.15D-01
5	P	4.05D-05	1.01D-05	5.95D-06	1.57D-06
	L	7.02D-05	1.31D-05	8.12D-06	4.14D-06
	Z	1.17D+00	1.78D-01	1.52D-01	1.38D-01
6	P	1.32D-06	5.24D-07	4.56D-07	4.58D-07
	L	1.29D-05	9.18D-07	6.64D-07	6.14D-07
	Z	6.32D-01	1.08D-01	9.62D-02	9.18D-02

Table 5.8.2-5

THE VARIANCE OF THE ESTIMABLES FOR THIS
SELECTION OF OBSERVATIONS

TYPE KEY- G INDICATES AN ANGLE
(UNITS FOR VARIANCE ARE ARC SECONDS SQUARED)
S INDICATES A DISTANCE
(UNITS FOR VARIANCE ARE METERS SQUARED)

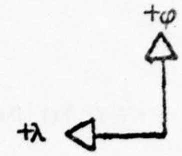
ESTIMABLE NUMBER/TYPE		VARIANCE INITIAL	VARIANCE ITERATION 1	VARIANCE ITERATION 2	VARIANCE ITERATION 3 (FINAL)
1	G	1.25D+00*	3.99D-01	2.21D-01	7.91D-02
2	G	1.87D-01	2.30D-02	2.09D-02	2.09D-02
3	G	5.85D+00*	3.85D-01	3.79D-01	3.72D-01
4	G	5.27D+00*	2.83D-01	2.78D-01	2.74D-01
5	G	6.35D-01	2.22D-01	1.66D-01	1.13D-01
6	G	5.05D+00*	2.27D-01	1.96D-01	1.94D-01
7	G	5.14D-01	5.09D-01	5.05D-01	5.01D-01
8	G	1.46D+00*	6.94D-01	5.99D-01	5.93D-01
9	G	1.13D+00*	8.22D-01	7.96D-01	7.94D-01
10	G	6.33D-01	3.81D-01	3.44D-01	3.43D-01
11	G	6.06D-01	1.76D-01	1.41D-01	1.19D-01
12	G	1.83D+00*	5.45D-01	3.55D-01	2.00D-01
13	G	3.47D+00*	1.52D-01	1.34D-01	1.33D-01
14	G	3.66D+00*	3.52D-01	3.08D-01	3.08D-01
15	G	3.59D+00*	2.51D-01	2.09D-01	2.08D-01
16	G	5.19D-01*	1.28D-01	1.26D-01	1.26D-01
17	S	1.64D-02*	2.90D-03	2.88D-03	2.62D-03
18	S	3.31D-03	2.54D-03	2.45D-03	2.45D-03
19	S	2.42D-02*	3.36D-03	3.15D-03	2.92D-03
20	S	6.24D-04	5.16D-04	5.14D-04	5.11D-04
21	S	6.93D-04	6.42D-04	6.38D-04	6.28D-04
22	S	5.42D-02*	1.44D-02*	8.31D-03*	2.07D-03
23	S	7.27D-04	7.01D-04	6.90D-04	6.62D-04
24	S	5.49D-04	3.61D-04	3.49D-04	3.49D-04
25	S	2.06D-03	1.26D-03	1.14D-03	1.13D-03
26	S	5.88D-04	5.74D-04	5.71D-04	5.65D-04
27	S	1.01D-01*	1.88D-02*	1.06D-02*	3.71D-03

(NOTE THAT "*" INDICATES ESTIMABLE FAILS USER REQUIREMENT)

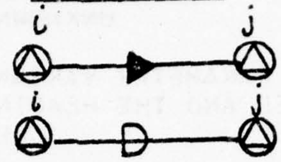
Table 5.8.2-5 (Continued)

Observations required in the design of example 5-2

NETWORK T 4



SYMBOL KEY



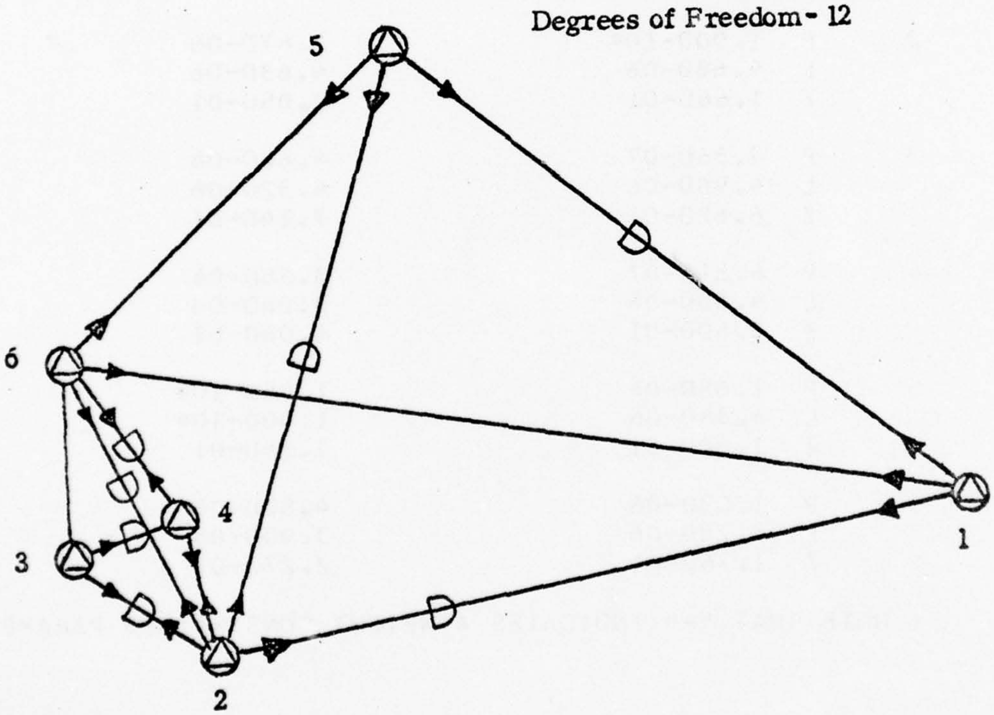
a direction from i to j

a distance between i and j

Number of Observations - 27

Rank of the Free Normals - 15

Degrees of Freedom - 12



Stations (△) joined by solid lines indicate that they are intervisible

Figure 5.8.2-1

STATION VARIANCES
FOR THIS NET

KEY TO PARAMETER CODE- P INDICATES THE VARIANCE IN LATITUDE
L INDICATES THE VARIANCE IN LONGITUDE
Z INDICATES THE VARIANCE IN STATION
UNKNOWN ('Z' CORRECTION)

NOTE THAT ALL PARAMETER VARIANCES ARE IN UNITS OF ARC SECONDS SQUARED AND THE HEADING 'CODE' INDICATES PARAMETER CODE

STATION NUMBER/CODE		VARIANCE MINIMUM CONSTRAINT	VARIANCE OVER CONSTRAINT
1	P	1.000D-10*	1.000D-10*
	L	1.000D-10*	1.000D-10*
	Z	1.170D-01	1.340D-01
2	P	1.000D-10*	2.670D-06
	L	4.680D-06	4.630D-06
	Z	1.660D-01	2.050D-01
3	P	9.560D-07	4.600D-06
	L	4.900D-06	4.320D-06
	Z	6.680D-01	7.140D-01
4	P	6.310D-07	3.360D-06
	L	4.630D-06	3.860D-06
	Z	3.600D-01	4.060D-01
5	P	1.680D-06	1.000D-10*
	L	6.380D-06	1.000D-10*
	Z	1.800D-01	1.650D-01
6	P	1.030D-06	4.500D-06
	L	5.280D-06	3.900D-06
	Z	1.760D-01	2.270D-01

(NOTE THAT "*" INDICATES A WEIGHT CONSTRAINED PARAMETER)

Table 5.8.2-6

THE VARIANCE OF THE ESTIMABLES FOR THIS
SELECTION OF OBSERVATIONS

TYPE KEY- G INDICATES AN ANGLE
(UNITS FOR VARIANCE ARE ARC SECONDS SQUARED)
S INDICATES A DISTANCE
(UNITS FOR VARIANCE ARE METERS SQUARED)

ESTIMABLE NUMBER/TYPE		VARIANCE MINIMUM CONSTRAINT	VARIANCE OVER CONSTRAINT
1	G	7.91D-02	6.24D-02
2	G	2.09D-02	1.84D-02
3	G	3.72D-01	3.52D-01
4	G	2.74D-01	2.51D-01
5	G	1.13D-01	5.59D-02
6	G	1.94D-01	1.69D-01
7	G	5.01D-01	5.01D-01
8	G	5.93D-01	5.92D-01
9	G	7.94D-01	7.94D-01
10	G	3.43D-01	3.43D-01
11	G	1.19D-01	1.07D-01
12	G	2.00D-01	1.86D-01
13	G	1.33D-01	1.21D-01
14	G	3.08D-01	2.93D-01
15	G	2.08D-01	1.96D-01
16	G	1.26D-01	8.74D-02
17	S	2.62D-03	2.36D-03
18	S	2.45D-03	1.44D-07
19	S	2.92D-03	2.16D-03
20	S	5.11D-04	5.11D-04
21	S	6.28D-04	6.28D-04
22	S	2.07D-03	2.07D-03
23	S	6.62D-04	6.60D-04
24	S	3.49D-04	3.49D-04
25	S	1.13D-03	1.13D-03
26	S	5.65D-04	5.64D-04
27	S	3.71D-03	3.66D-03

Table 5.8.2-6 (Continued)

DESCRIPTION OF THE OBSERVATIONS

KEY - D INDICATES A DIRECTION
(UNITS FOR VARIANCE ARE ARC SECONDS SQUARED)

A INDICATES AN AZIMUTH
(UNITS FOR VARIANCE ARE ARC SECONDS SQUARED)

S INDICATES A DISTANCE
(UNITS FOR VARIANCE ARE METERS SQUARED)

OBSERVATION NUMBER	TYPE	A PRIORI VARIANCE	FROM STATION	TO STATION
1	D	0.160	1	2
2	D	0.160	1	3
3	D	0.160	1	4
4	D	0.160	2	1
5	D	0.160	2	3
6	D	0.160	2	4
7	D	0.160	3	1
8	D	0.160	3	2
9	D	0.160	3	4
10	D	0.160	3	5
11	D	0.160	3	6
12	D	0.160	4	1
13	D	0.160	4	2
14	D	0.160	4	3
15	D	0.160	4	5
16	D	0.160	4	6
17	D	0.160	4	7
18	D	0.160	5	3
18	D	0.160	5	4
20	D	0.160	5	6
21	D	0.160	5	7
22	D	0.160	6	3
23	D	0.160	6	4
24	D	0.160	6	5
25	D	0.160	6	7
26	D	0.160	6	8
27	D	0.160	6	9
28	D	0.160	7	4
29	D	0.160	7	5
30	D	0.160	7	6
31	D	0.160	7	8
32	D	0.160	7	9
33	D	0.160	8	6
34	D	0.160	8	7
35	D	0.160	8	9

Table 5.8.3-I

DESCRIPTION OF THE OBSERVATIONS
(CONTINUED)

OBSERVATION NUMBER	TYPE	A PRIORI VARIANCE	FROM STATION	TO STATION
36	D	0.160	9	6
37	D	0.160	9	7
38	D	0.160	9	8
39	S	0.113X10**-2	1	2
40	S	0.116X10**-2	1	3
41	S	0.118X10**-2	1	4
42	S	0.968X10**-3	2	3
43	S	0.205X10**-2	2	4
44	S	0.122X10**-2	3	4
45	S	0.109X10**-2	3	5
46	S	0.112X10**-2	3	6
47	S	0.866X10**-3	4	5
48	S	0.137X10**-2	4	6
49	S	0.117X10**-2	4	7
50	S	0.629X10**-3	5	6
51	S	0.836X10**-3	5	7
52	S	0.115X10**-2	6	7
53	S	0.158X10**-2	6	8
54	S	0.954X10**-3	6	9
55	S	0.937X10**-3	7	8
56	S	0.136X10**-2	7	9
57	S	0.107X10**-2	8	9

Table 5.8.3-1 (Continued)

AD-A033 671 DEFENSE MAPPING AGENCY WASHINGTON D C F/G 8/5

THE DESIGN OF SPECIAL PURPOSE HORIZONTAL GEODETIC CONTROL NETWO--ETC(U)

OCT 76 W H SPRINSKY

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3 OF 3

ADA033671

END

DATE

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2 -77

DESCRIPTION OF THE ESTIMABLES IN THIS TEST

KEY - A INDICATES AN AZIMUTH
(UNITS FOR VARIANCE ARE ARC SECONDS SQUARED)
S INDICATES A DISTANCE
(UNITS FOR VARIANCE ARE METERS SQUARED)
G INDICATES AN ANGLE
(UNITS FOR VARIANCE ARE ARC SECONDS SQUARED)

ESTIMABLE NUMBER	TYPE	TARGET VARIANCE	AT STATION	FROM STATION	TO STATION
1	G	0.70	1	2	3
2	G	0.70	1	2	4
3	G	0.70	2	1	3
4	G	0.70	2	1	4
5	G	0.70	3	1	2
6	G	0.70	3	1	4
7	G	0.70	3	1	5
8	G	0.70	3	1	6
9	G	0.70	4	1	2
10	G	0.70	4	1	3
11	G	0.70	4	1	5
12	G	0.70	4	1	6
13	G	0.70	4	1	7
14	G	0.70	5	3	4
15	G	0.70	5	3	6
16	G	0.70	5	3	7
17	G	0.70	6	3	4
18	G	0.70	6	3	5
19	G	0.70	6	3	7
20	G	0.70	6	3	8
21	G	0.70	6	3	9
22	G	0.70	7	4	5
23	G	0.70	7	4	6
24	G	0.70	7	4	8
25	G	0.70	7	4	9
26	G	0.70	8	6	7
27	G	0.70	8	6	9
28	G	0.70	9	6	7
29	G	0.70	9	6	8

Table 5.8.3-2

DESCRIPTION OF THE ESTIMABLES IN THIS TEST
(CONTINUED)

ESTIMABLE NUMBER	TYPE	TARGET VARIANCE	FROM	TO
30	S	0.386X10**-2	1	2
31	S	0.393X10**-2	1	3
32	S	0.401X10**-2	1	4
33	S	0.320X10**-2	2	3
34	S	0.696X10**-2	2	4
35	S	0.413X10**-2	3	4
36	S	0.372X10**-2	3	5
37	S	0.382X10**-2	3	6
38	S	0.295X10**-2	4	5
39	S	0.467X10**-2	4	6
40	S	0.398X10**-2	4	7
41	S	0.213X10**-2	5	6
42	S	0.284X10**-2	5	7
43	S	0.392X10**-2	6	7
44	S	0.538X10**-2	6	8
45	S	0.324X10**-2	6	9
46	S	0.319X10**-2	7	8
47	S	0.462X10**-2	7	9
48	S	0.365X10**-2	8	9

Table 5.8.3-2 (Continued)

LOCATION OF COEFFICIENTS IN THE V/C MATRIX OF PARAMETERS WHICH ARE UNKNOWNS

EXPLANATION OF TABLE - SYMBOLS AT THE START OF EACH ROW INDICATE THE UNKNOWN PARAMETER TO WHICH THAT ROW AND COLUMN REFER

A ZERO INDICATES THAT THIS COEFFICIENT IS CONSTRAINED AT A VALUE OF ZERO AND IS NOT AN UNKNOWN (DECOUPLED)

ϕ_1	1	2	3	4	5	6	7	0	0	0	0	0	0	0	0	0	0
	λ_1	8	9	10	11	12	13	0	0	0	0	0	0	0	0	0	0
		ϕ_2	14	15	16	17	18	0	0	0	0	0	0	0	0	0	0
			λ_2	19	20	21	22	0	0	0	0	0	0	0	0	0	0
				ϕ_3	23	24	25	26	27	28	26	0	0	0	0	0	0
					λ_3	30	31	32	33	34	35	0	0	0	0	0	0
						ϕ_4	36	37	38	39	40	41	42	0	0	0	0
							λ_4	43	44	45	46	47	48	0	0	0	0
								ϕ_5	49	50	51	52	53	0	0	0	0
									λ_5	54	55	56	57	0	0	0	0
										ϕ_6	58	59	60	61	62	63	64
											λ_6	65	66	67	68	69	70
												ϕ_7	71	72	73	74	75
													λ_7	76	77	78	79
														ϕ_8	80	81	82
															λ_8	83	84
																ϕ_9	85
																	λ_9

Table 5.8.3-3

SOLUTION FOR THE OFF-DIAGONAL COEFFICIENTS
IN THE V/C MATRIX OF PARAMETERS (X_C)

COEFFICIENT NUMBER	A SOLUTION VALUE	B SOLUTION VALUE
1	6.8197784E-07	5.5205862E-07
2	1.2588714E-07	-3.3573724E-07
3	-1.3621360E-07	1.1381326E-06
4	2.4039178E-07	-4.2621650E-07
5	-9.4843188E-08	1.0738968E-06
6	-5.1276356E-07	-2.0592870E-07
7	-6.4747877E-07	1.0263548E-06
8	1.7224750E-07	-7.1357681E-07
9	-7.4849288E-07	-2.8325758E-07
10	4.1503796E-07	-1.2497803E-06
11	-7.1270449E-08	6.9203452E-09
12	-6.8366199E-07	-7.8520225E-07
13	2.0376018E-07	1.0669992E-06
14	3.3035182E-08	1.2011069E-06
15	2.5858606E-07	-1.9843401E-07
16	-2.4931273E-07	4.3200816E-07
17	-1.5801709E-07	-1.3496401E-06
18	-1.0803285E-07	9.7127668E-08
19	-4.4020328E-07	-7.0982333E-07
20	5.8768404E-07	1.8231167E-06
21	4.3356499E-07	6.5999416E-08
22	3.9040265E-07	7.8398989E-07
23	5.4049030E-07	-2.1743290E-07
24	-2.4701058E-07	-8.0529662E-07
25	3.7243268E-08	1.7562525E-07
26	2.1888013E-07	4.8465637E-07
27	-5.9958029E-07	-1.4808109E-07
28	2.3923326E-07	5.2845724E-07
29	-2.0214162E-07	3.0622687E-07
30	-3.4280527E-07	-4.5637609E-07
31	-3.7833394E-07	5.3141866E-07
32	-1.3040898E-07	2.9699089E-07
33	7.4676257E-07	6.3979223E-07
34	-3.0616104E-07	6.2605341E-07
35	1.3098543E-06	6.8422833E-07

Table 5.8.3-4

SOLUTION FOR THE OFF-DIAGONAL COEFFICIENTS
(CONTINUED)

COEFFICIENT NUMBER	A SOLUTION VALUE	B SOLUTION VALUE
36	-1.8880048E-08	1.1265975E-06
37	2.5789353E-07	3.0273907E-07
38	3.4303139E-07	-1.5341925E-08
39	-4.7104578E-07	5.4670727E-07
40	2.6426017E-07	-1.2838063E-07
41	3.4394836E-09	-1.7566890E-07
42	-3.9345807E-08	3.4932117E-07
43	9.4852095E-07	1.6885479E-06
44	8.9980989E-07	1.4907364E-06
45	1.8865546E-07	1.4509778E-06
46	1.4110788E-07	8.3615123E-07
47	-9.2082644E-08	6.4735673E-07
48	1.2495202E-06	7.1104114E-07
49	-4.3137181E-08	1.0935373E-06
50	9.2452410E-07	1.1637030E-06
51	1.3018973E-07	2.6190173E-07
52	6.8964187E-07	6.7977624E-07
53	-4.7659893E-07	1.1871462E-06
54	6.2467302E-07	1.6142867E-06
55	1.4893732E-07	1.4976686E-06
56	-5.7264589E-07	3.6802430E-07
57	3.9378307E-07	1.4309180E-06
58	-3.5572481E-07	8.2001679E-07
59	2.2518048E-08	4.0629857E-08
60	-7.0313533E-09	1.0700214E-06
61	-3.7465611E-08	-1.4417001E-06
62	-1.8378591E-07	-1.4074976E-07
63	-2.5413550E-07	-1.2285127E-06
64	8.5632053E-08	2.5917961E-07
65	-4.6043522E-07	-3.1671334E-07
66	-2.5175893E-07	1.5853002E-06
67	3.6093303E-07	-9.0433787E-08
68	1.5586210E-07	1.4480320E-06
69	1.3211786E-07	1.3482841E-06
70	1.1537441E-06	1.7023240E-06
71	3.3786830E-07	1.0196363E-06
72	-7.7685058E-08	-1.1728462E-06
73	9.8944383E-08	1.2076589E-06
74	4.3086771E-08	-5.8749174E-07
75	1.1740065E-07	-1.4863235E-07

Table 5.8.3-4(Continued)

SOLUTION FOR THE OFF-DIAGONAL COEFFICIENTS
(CONTINUED)

COEFFICIENT NUMBER	A SOLUTION VALUE	B SOLUTION VALUE
76	-1.0780332E-07	-1.1570319E-07
77	1.3845038E-06	1.6668291E-06
78	1.4176237E-07	1.1304965E-06
79	9.6376255E-08	8.2483166E-07
80	2.2124601E-07	-5.8635669E-07
81	4.1227338E-08	-4.5583874E-07
82	-3.5278703E-07	-1.7015991E-06
83	-1.8619733E-07	4.4000626E-07
84	1.1131232E-07	-3.2005482E-07
85	8.6854016E-08	6.7311339E-07

Table 5.8.3-4 (Continued)

PREDICTED VARIANCES FOR ESTIMABLES
USING CRITERIAN V/C MATRIX OF PARAMETERS

ESTIMABLE NUMBER	A SOLUTION ESTIMABLE VARIANCE	B SOLUTION ESTIMABLE VARIANCE
1	6.22D-01	6.18D-01
2	7.00D-01	7.77D-01
3	7.00D-01	6.73D-01
4	4.02D-01	3.55D-01
5	7.00D-01	6.92D-01
6	7.00D-01	6.86D-01
7	7.00D-01	6.26D-01
8	7.00D-01	7.06D-01
9	3.81D-01	3.31D-01
10	6.23D-01	6.40D-01
11	7.00D-01	7.78D-01
12	7.00D-01	5.16D-01
13	7.00D-01	6.96D-01
14	7.00D-01	6.63D-01
15	7.00D-01	7.14D-01
16	7.00D-01	7.59D-01
17	5.95D-01	4.85D-01
18	7.00D-01	7.18D-01
19	7.00D-01	6.86D-01
20	7.00D-01	6.67D-01
21	7.00D-01	6.88D-01
22	7.00D-01	5.55D-01
23	7.00D-01	7.57D-01
24	7.00D-01	7.09D-01
25	7.00D-01	7.09D-01
26	6.79D-01	6.21D-01
27	4.84D-01	5.06D-01
28	7.00D-01	6.54D-01
29	7.00D-01	6.96D-01
30	4.01D-03	3.54D-03

Table 5.8.3-5/6

PREDICTED VARIANCES FOR ESTIMABLES
USING CRITERIAN V/C MATRIX OF PARAMETERS
(CONTINUED)

ESTIMABLE NUMBER	A SOLUTION ESTIMABLE VARIANCE	B SOLUTION ESTIMABLE VARIANCE
31	2.22D-03	3.14D-03
32	4.82D-03	3.93D-03
33	3.16D-03	3.74D-03
34	2.71D-03	5.00D-03
35	3.80D-03	2.99D-03
36	3.36D-03	2.71D-03
37	2.69D-03	1.99D-03
38	3.34D-03	1.63D-03
39	3.93D-03	1.67D-03
40	3.06D-03	3.44D-03
41	3.30D-03	1.33D-03
42	3.23D-03	2.02D-03
43	3.53D-03	1.88D-03
44	2.83D-03	4.51D-03
45	3.58D-03	5.36D-03
46	3.00D-03	5.32D-03
47	2.89D-03	2.19D-03
48	3.29D-03	3.98D-03

Table 5.8.3-5/6 (Continued)

ABSTRACT OF PREDICTED OBSERVATION CORRELATION

EXPLANATION OF TABLE- COLUMN A IS THE OBSERVATION BEING CONSIDERED

COLUMN B ARE THE OTHER OBSERVATIONS WITH WHICH THE COLUMN A OBSERVATION IS CORRELATED WITH COEFFICIENTS OF BETWEEN 1. AND .80

COLUMN C ARE THE OTHER OBSERVATIONS WITH WHICH THE COLUMN A OBSERVATION IS CORRELATED WITH COEFFICIENTS OF BETWEEN .79 AND .50

COLUMN A	COLUMN B	COLUMN C
1	NONE	2, 3, 4
2	NONE	1, 3, 7
3	NONE	2, 1
4	6	5, 1
5	6	4
6	4, 5	NONE
7	NONE	9, 8, 10, 2, 11
8	NONE	7, 9, 10, 11
9	NONE	7, 10, 8, 11, 14, 46
10	11	9, 7, 8
11	10	9, 7, 8
12	NONE	13, 14, 16, 15, 17
13	14, 15, 16	12, 17
14	13, 16	15, 12, 9, 17, 46
15	13, 16	14, 17, 12, 19
16	13, 14, 15, 17	12

Table 5.8.3-7

ABSTRACT OF PREDICTED OBSERVATION
CORRELATION (CONTINUED)

COLUMN A	COLUMN B	COLUMN C
*****	*************************	*************************
17	16	15, 13, 12, 14
18	NONE	19, 20, 21
19	NONE	18, 15
20	NONE	18, 21, 24, 47, 52, 53
21	NONE	18, 20
22	NONE	23, 24, 25, 26, 27
23	NONE	22, 24, 25, 26, 27
24	NONE	23, 25, 22, 26, 20, 47, 52, 53
25	NONE	23, 24, 22, 26
26	NONE	23, 24, 25, 27, 22, 55
27	NONE	26, 22, 23
28	NONE	29, 30, 31, 32
29	NONE	30, 32, 28
30	NONE	29, 32, 28
31	NONE	28, 32
32	NONE	29, 30, 28, 31, 48, 50
33	NONE	35, 34, 55
34	NONE	33
35	NONE	33, 38
36	NONE	37, 38
37	NONE	36, 38, 48, 50

Table 5.8.3-7 (Continued)

ABSTRACT OF PREDICTED OBSERVATION
CORRELATION (CONTINUED)

COLUMN A	COLUMN B	COLUMN C
*****	*************************	*************************
38	NONE	36, 37, 35
39	NONE	40
40	NONE	39
41	NONE	43, 44
42	NONE	43
43	NONE	44, 41, 42
44	NONE	43, 45, 41
45	NONE	46, 44, 50
46	NONE	45, 9, 14, 50
47	NONE	48, 20, 24
48	NONE	47, 50, 32, 37
49	NONE	NONE.
50	NONE	48, 32, 37, 45, 46
51	NONE	52
52	NONE	51, 20, 24
53	NONE	54, 20, 24, 57
54	NONE	53
55	NONE	26, 33
56	NONE	NONE
57	NONE	53

Table 5.8.3-7 (Continued)

SELECTION OF OBSERVATIONS FOR THE FORMATION OF THE V/C MATRIX FOR POSITIONAL PARAMETERS

EXPLANATION OF THE ENTRIES IN THE 'SELECTION' COLUMN

I - INDICATES A SELECTION (INITIAL FORMATION)

A - INDICATES AN ADDITION. THE NUMBER FOLLOWING IS THE ITERATION IN WHICH THIS OBSERVATION WAS ADDED

KEPT - INDICATES THAT THIS OBSERVATION WAS INCLUDED TO INSURE THAT AT LEAST TWO DIRECTIONS WERE OBSERVED AT EACH STATION

.J - WHERE J IS A NUMBER FROM 1 TO 9 INDICATES THAT THIS OBSERVATION WAS CORRELATED TO ANOTHER SELECTED OBSERVATION WITH A COEFFICIENT OF .J

(NOTE - THOSE OBSERVATIONS MARKED 'ELIMINATED ALT 1 OR 2 INDICATE THAT OBSERVATION WAS INCLUDED IN FIRST ITERATION OR THE INITIAL SELECTION AND REMOVED IN ALTERNATE CHOICE 1 OR ALTERNATE CHOICE 2)

OBSERVATION NUMBER	SELECTION	OBSERVATION NUMBER	SELECTION
1	I	21	I
2 .7		22	I
3 KEPT		23 .7	
4	I	24 .6	
5 KEPT		25 .6	
6 .8		26 .6	
7	I	27KEPT	
8 .6		28	I
9 .7		29 .6	
10 .6		30KEPT	
11	I	31 .6	A1,ELIMINATED IN ALT 1 & 2
12	I	32 .6	A1
13 .7		33	I
14 .6		34KEPT	
15 .6		35 .7	
16 .6		36	I
17	I	37 .6	
18	I	38KEPT	
19 .6		39	I
20 .6	A1		

Table 5.8.3-8

SELECTION OF OBSERVATIONS FOR THE FORMATION OF THE V/C
MATRIX FOR POSITIONAL PARAMETERS
(CONTINUED)

(NOTE - THOSE OBSERVATIONS MARKED 'ELIMINATED ALT 1 OR 2 INDICATE THAT THE OBSERVATION WAS INCLUDED IN THE FIRST AND OR THE INITIAL SELECTION AND REMOVED IN ALTERNATE CHOICE 1 OR ALTERNATE CHOICE 2)

OBSERVATION NUMBER	SELECTION	OBSERVATION NUMBER	SELECTION
40 .6		50 .6	
41	I	51	I
42	I	52 .6	
43 .6		53	I
44	I	54 .6	A1
45 .6		55	I, ELIMINATED ALT 1
46	I	56	I
47	I	57 .6	
48 .6			
49	I		
50 .6			

Table 5.8.3-8 (Continued)

STATION VARIANCES
FOR THIS NET

KEY TO PARAMETER CODE- P INDICATES THE VARIANCE IN LATITUDE
L INDICATES THE VARIANCE IN LONGITUDE
Z INDICATES THE VARIANCE IN STATION
UNKNOWN ('Z' CORRECTION)

NOTE THAT ALL PARAMETER VARIANCES ARE IN UNITS OF ARC SECONDS SQUARED AND THE HEADING 'CODE' INDICATES PARAMETER CODE

STATION NUMBER	CODE	VARIANCE INITIAL	VARIANCE ITERATION 1	VARIANCE ALTERNATE 1	VARIANCE ALTERNATE 2 (FINAL)
1	P	6.42D-07	6.86D-07	6.77D-07	6.86D-07
	L	8.02D-07	8.54D-07	8.82D-07	8.52D-07
	Z	1.48D-01	1.54D-01	1.58D-01	1.52D-01
2	P	1.05D-06	1.20D-06	1.18D-06	1.18D-06
	L	1.20D-06	1.28D-06	1.29D-06	1.26D-06
	Z	2.63D-01	2.74D-01	2.77D-01	2.70D-01
3	P	6.12D-07	5.62D-07	5.45D-07	5.59D-07
	L	4.85D-07	4.44D-07	4.37D-07	4.36D-07
	Z	1.24D-01	1.21D-01	1.26D-01	1.20D-01
4	P	4.31D-07	4.68D-07	5.40D-07	4.72D-07
	L	4.73D-07	4.62D-07	4.64D-07	4.64D-07
	Z	1.10D-01	9.81D-02	1.06D-01	1.01D-01
5	P	7.04D-07	2.17D-07	2.36D-07	2.18D-07
	L	1.25D-06	6.67D-07	7.36D-07	6.81D-07
	Z	1.33D-01	8.86D-02	1.04D-01	9.01D-02
6	P	8.11D-07	2.25D-07	2.32D-07	2.29D-07
	L	8.81D-07	6.06D-07	6.51D-07	5.78D-07
	Z	2.07D-01	1.14D-01	1.15D-01	1.14D-01
7	P	3.16D-07	2.59D-07	4.37D-07	2.64D-07
	L	7.74D-07	2.81D-07	4.50D-07	3.78D-07
	Z	1.62D-01	6.65D-02	9.94D-02	8.30D-02
8	P	6.80D-07	5.49D-07	1.92D-06	5.55D-07
	L	5.35D-06	9.78D-07	4.93D-06	2.99D-06
	Z	5.48D-01	1.81D-01	5.70D-01	3.56D-01
9	P	7.97D-06	6.14D-07	7.12D-07	6.46D-07
	L	2.73D-06	7.84D-07	7.60D-07	7.57D-07
	Z	4.96D-01	1.68D-01	2.54D-01	1.71D-01

Table 5.8.3-9

THE VARIANCE OF THE ESTIMABLES FOR THIS
SELECTION OF OBSERVATIONS

TYPE KEY- G INDICATES AN ANGLE
(UNITS FOR VARIANCE ARE ARC SECONDS SQUARED)
S INDICATES A DISTANCE
(UNITS FOR VARIANCE ARE METERS SQUARED)

ESTIMABLE NUMBER/TYPE		VARIANCE INITIAL	VARIANCE ITERATION 1	VARIANCE ALTERNATE 1	VARIANCE ALTERNATE 2 (FINAL)
1	G	1.44D-01	1.43D-01	1.43D-01	1.43D-01
2	G	2.01D-01	1.98D-01	1.98D-01	1.98D-01
3	G	2.18D-01	2.17D-01	2.18D-01	2.18D-01
4	G	7.33D-02	7.19D-02	7.19D-02	7.19D-02
5	G	3.12D-01	3.10D-01	3.11D-01	3.11D-01
6	G	5.78D-01	5.66D-01	5.67D-01	5.67D-01
7	G	5.66D-01	5.53D-01	5.53D-01	5.53D-01
8	G	5.27D-01	4.96D-01	4.97D-01	4.96D-01
9	G	8.78D-02	8.60D-02	8.63D-02	8.61D-02
10	G	2.17D-01	2.06D-01	2.07D-01	2.06D-01
11	G	5.15D-01	3.99D-01	4.01D-01	3.99D-01
12	G	2.50D-01	2.11D-01	2.13D-01	2.11D-01
13	G	2.00D-01	1.89D-01	1.90D-01	1.90D-01
14	G	4.39D-01	3.30D-01	3.32D-01	3.30D-01
15	G	1.29D+00*	2.50D-01	2.55D-01	2.50D-01
16	G	2.62D-01	2.56D-01	2.57D-01	2.56D-01
17	G	1.82D-01	1.20D-01	1.24D-01	1.20D-01
18	G	1.27D+00*	2.56D-01	2.64D-01	2.57D-01
19	G	4.50D-01	2.25D-01	2.35D-01	2.26D-01
20	G	7.99D-01	2.93D-01	6.72D-01	3.90D-01
21	G	3.18D-01	2.34D-01	2.69D-01	2.35D-01
22	G	1.39D-01	1.31D-01	1.34D-01	1.32D-01
23	G	2.23D-01	1.34D-01	1.44D-01	1.37D-01
24	G	7.50D-01	2.15D-01	8.86D-01	6.01D-01
25	G	1.35D-01	1.63D-01	1.67D-01	1.64D-01
26	G	2.19D-01	9.81D-02	1.80D-01	1.80D-01
27	G	1.49D+00*	1.05D-01	1.38D-01	1.38D-01
28	G	3.83D-01	1.09D-01	1.75D-01	1.10D-01
29	G	2.67D-01	1.93D-01	3.18D-01	2.04D-01
30	S	8.51D-04	8.49D-04	8.50D-04	8.50D-04

(NOTE THAT "*" INDICATES ESTIMABLE FAILS USER REQUIREMENT)

Table 5.8.3-9 (Continued)

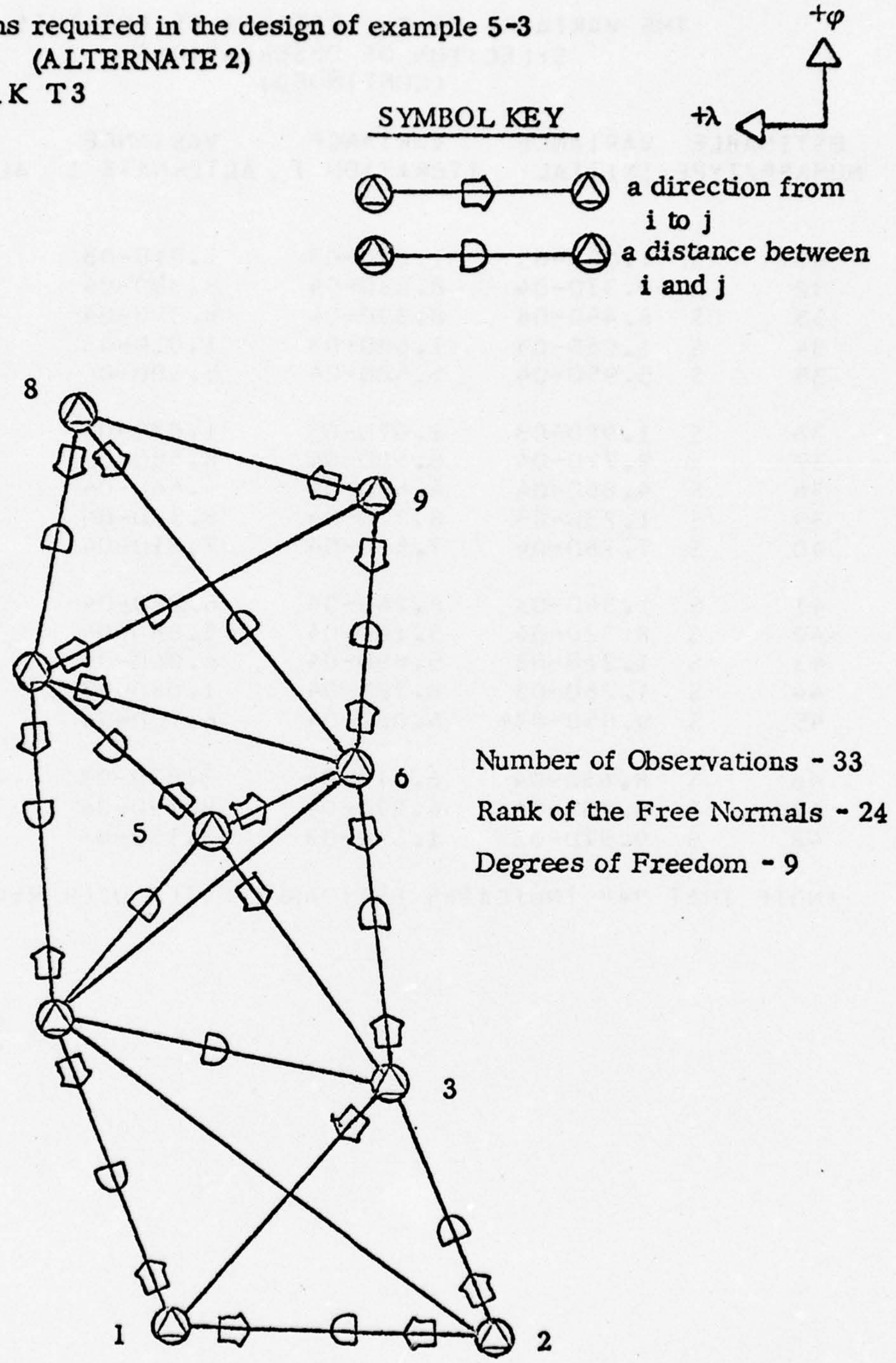
THE VARIANCE OF THE ESTIMABLES FOR THIS
SELECTION OF OBSERVATIONS
(CONTINUED)

ESTIMABLE NUMBER/TYPE		VARIANCE INITIAL	VARIANCE ITERATION 1	VARIANCE ALTERNATE 1	VARIANCE ALTERNATE 2 (FINAL)
31	S	1.02D-03	1.01D-03	1.01D-03	1.01D-03
32	S	9.37D-04	8.83D-04	8.89D-04	8.84D-04
33	S	8.44D-04	8.39D-04	8.39D-04	8.39D-04
34	S	1.05D-03	1.00D-03	1.01D-03	1.00D-03
35	S	5.95D-04	5.50D-04	5.60D-04	5.50D-04
36	S	1.98D-03	1.07D-03	1.07D-03	1.07D-03
37	S	9.72D-04	8.56D-04	8.58D-04	8.56D-04
38	S	4.86D-04	4.66D-04	4.66D-04	4.66D-04
39	S	1.23D-03	8.32D-04	8.38D-04	8.35D-04
40	S	7.76D-04	7.51D-04	7.61D-04	7.53D-04
41	S	1.34D-03	8.24D-04	8.27D-04	8.25D-04
42	S	8.32D-04	5.14D-04	5.38D-04	5.17D-04
43	S	1.21D-03	5.88D-04	6.06D-04	5.91D-04
44	S	1.26D-03	6.78D-04	1.08D-03	1.08D-03
45	S	9.85D-03*	6.03D-04	6.70D-04	6.32D-04
46	S	8.65D-04	6.51D-04	3.57D-03	7.42D-04
47	S	8.25D-04	6.97D-04	9.79D-04	7.13D-04
48	S	9.97D-03*	1.34D-03	3.13D-03	2.63D-03

(NOTE THAT "*" INDICATES ESTIMABLE FAILS USER REQUIREMENT)

Table 5.8.3-9 (Continued)

Observations required in the design of example 5-3
(ALTERNATE 2)
NETWORK T3



Stations ( ) joined by solid lines indicate that they are intervisible

Figure 5.8.3-1

STATION VARIANCES
FOR THIS NET

KEY TO PARAMETER CODE- P INDICATES THE VARIANCE IN LATITUDE
L INDICATES THE VARIANCE IN LONGITUDE
Z INDICATES THE VARIANCE IN STATION UNKNOWN ('Z' CORRECTION)
'*' INDICATES THAT THIS PARAMETER HAS BEEN WEIGHT CONSTRAINED

NOTE THAT ALL PARAMETER VARIANCES ARE IN UNITS OF ARC SECONDS SQUARED AND THE HEADING 'CODE' INDICATES PARAMETER CODE

STATION NUMBER/CODE		VARIANCE MINIMUM CONSTRAINT 1	VARIANCE MINIMUM CONSTRAINT 2	VARIANCE OVER CONSTRAINT 1	VARIANCE OVER CONSTRAINT 2
1	P	1.00D-10*	1.00D-10*	1.00D-10*	1.00D-10*
	L	1.00D-10*	1.00D-10*	1.00D-10*	1.00D-10*
	Z	1.27D-01	1.67D-01	8.00D-02	9.48D-02
2	P	1.00D-10*	1.30D-06	1.00D-10*	1.00D-10*
	L	1.52D-06	1.58D-06	1.00D-10*	1.00D-10*
	Z	1.37D-01	2.91D-01	1.03D-01	1.14D-01
3	P	9.36D-07	1.02D-06	5.40D-07	6.33D-07
	L	1.91D-06	1.30D-06	6.03D-07	9.02D-07
	Z	2.31D-01	1.14D-01	9.49D-02	1.04D-01
4	P	1.14D-06	9.22D-07	1.00D-10*	6.31D-06
	L	2.09D-06	1.00D-06	1.00D-10*	7.71D-07
	Z	2.68D-01	1.06D-01	9.72D-02	1.00D-01
5	P	1.21D-06	1.24D-06	3.22D-07	4.64D-07
	L	5.48D-06	1.43D-05	8.01D-07	1.18D-06
	Z	3.01D-01	1.38D-01	1.34D-01	1.17D-01
6	P	1.41D-06	1.28D-06	1.00D-10*	3.74D-07
	L	6.48D-06	9.64D-07	1.00D-10*	6.78D-07
	Z	3.45D-01	1.16D-01	1.61D-01	1.04D-01
7	P	1.80D-06	1.44D-06	6.53D-07	4.91D-07
	L	8.42D-06	1.12D-06	8.36D-07	8.65D-07
	Z	3.20D-01	1.18D-01	1.18D-01	9.07D-02
8	P	2.01D-06	1.79D-06	1.00D-06	1.00D-10*
	L	2.11D-05	5.22D-06	5.97D-06	1.00D-10*
	Z	6.64D-01	4.62D-01	4.54D-01	1.24D-01
9	P	2.10D-06	1.75D-06	6.77D-07	1.00D-10*
	L	1.47D-05	1.00D-10*	1.56D-06	1.00D-10*
	Z	4.47D-01	2.10D-01	2.55D-01	1.00D-01

Table 5.8.3-10

THE VARIANCE OF THE ESTIMABLES FOR THIS
SELECTION OF OBSERVATIONS

TYPE KEY- G INDICATES AN ANGLE
(UNITS FOR VARIANCE ARE ARC SECONDS SQUARED)
S INDICATES A DISTANCE
(UNITS FOR VARIANCE ARE METERS SQUARED)

ESTIMABLE NUMBER/TYPE		VARIANCE MINIMUM CONSTRAINT 1	VARIANCE MINIMUM CONSTRAINT 2	VARIANCE OVER CONSTRAINT 1	VARIANCE OVER CONSTRAINT 2
1	G	1.43D-01	1.43D-01	4.41D-02	5.58D-02
2	G	1.98D-01	1.98D-01	5.62D-05	5.91D-02
3	G	2.18D-01	2.18D-01	9.36D-02	1.34D-01
4	G	7.19D-02	7.19D-02	2.37D-05	2.61D-02
5	G	3.11D-01	3.11D-01	1.17D-01	1.43D-01
6	G	5.67D-01	5.67D-01	2.90D-01	4.27D-01
7	G	5.53D-01	5.53D-01	3.48D-01	3.78D-01
8	G	4.96D-01	4.96D-01	2.91D-01	3.11D-01
9	G	8.61D-02	8.61D-02	1.58D-05	2.32D-02
10	G	2.06D-01	2.06D-01	7.39D-02	1.74D-01
11	G	3.99D-01	3.99D-01	1.06D-01	3.15D-01
12	G	2.11D-01	2.11D-01	3.59D-05	1.63D-01
13	G	1.90D-01	1.90D-01	6.89D-02	1.58D-01
14	G	3.30D-01	3.30D-01	2.26D-01	2.72D-01
15	G	2.50D-01	2.50D-01	2.40D-01	2.49D-01
16	G	2.56D-01	2.56D-01	2.43D-01	2.52D-01
17	G	1.20D-01	1.20D-01	5.68D-02	1.04D-01
18	G	2.57D-01	2.57D-01	2.09D-01	2.29D-01
19	G	2.26D-01	2.26D-01	1.22D-01	1.67D-01
20	G	3.90D-01	3.90D-01	2.72D-01	1.20D-01
21	G	2.35D-01	2.35D-01	1.86D-01	1.95D-01
22	G	1.32D-01	1.32D-01	1.23D-01	1.21D-01
23	G	1.37D-01	1.37D-01	7.78D-02	1.11D-01
24	G	6.01D-01	6.01D-01	5.76D-01	2.44D-01
25	G	1.64D-01	1.64D-01	1.56D-01	1.12D-01
26	G	1.80D-01	1.80D-01	1.63D-01	1.09D-01
27	G	1.38D-01	1.38D-01	1.29D-01	3.47D-02
28	G	1.10D-01	1.10D-01	9.83D-02	9.76D-02
29	G	2.04D-01	2.04D-01	2.01D-01	8.08D-02
30	S	8.50D-04	8.50D-04	1.12D-07	1.12D-07

Table 5.8.3-10 (Continued)

THE VARIANCE OF THE ESTIMABLES FOR THIS
SELECTION OF OBSERVATIONS
(CONTINUED)

ESTIMABLE NUMBER/TYPE		VARIANCE INITIAL	VARIANCE 1 ITERATION	VARIANCE 2 ITERATION (FINAL)	VARIANCE MINIMUM CONSTRAINT
31	S	1.01D-03	1.01D-03	5.49D-04	7.25D-04
32	S	8.84D-04	8.84D-04	1.81D-07	6.14D-04
33	S	8.39D-04	8.39D-04	4.12D-04	4.77D-04
34	S	1.00D-03	1.00D-03	1.40D-07	5.42D-04
35	S	5.50D-04	5.50D-04	2.95D-04	5.14D-04
36	S	1.07D-03	1.07D-03	7.60D-04	7.65D-04
37	S	8.56D-04	8.56D-04	4.86D-04	5.68D-04
38	S	4.66D-04	4.66D-04	3.68D-04	4.30D-04
39	S	8.35D-04	8.35D-04	1.42D-07	6.17D-04
40	S	7.53D-04	7.53D-04	6.06D-04	5.73D-04
41	S	8.25D-04	8.25D-04	4.23D-04	6.79D-04
42	S	5.17D-04	5.17D-04	4.63D-04	5.10D-04
43	S	5.91D-04	5.91D-04	3.81D-04	5.31D-04
44	S	1.08D-03	1.08D-03	1.02D-03	3.10D-04
45	S	6.32D-04	6.32D-04	6.11D-04	3.66D-04
46	S	7.42D-04	7.42D-04	7.38D-04	4.82D-04
47	S	7.13D-04	7.13D-04	5.62D-04	5.11D-04
48	S	2.63D-03	2.63D-03	2.43D-03	1.16D-07

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Table 5.8.3-10 (Continued)

6. The Weight Recovery Method

6.1 An alternate approach to design.

As mentioned previously, a solution to the equation referred to as Theorem B, Bossler et al. (1973), provides another method of selection of observations. Prior to examination of this equation, some initial discussion is required. In previous chapters, the inverses of the normal equations were discussed. The formation of the normals from the (inconsistent) observation equations implies the (first) minimization of the weighted sum of the squares of the corrections to the observations (the residuals). If the rank of the normal equation matrix is less than its order, a second minimization is performed, the quantity minimized being the sum of the squares of the parameters or corrections to the initial values of the parameters, the $X'X$ minimum solution. If the observation equations are solved directly, a solution which does both these minimizations is the generalized inverse of the observation design matrix which solves for the unknown as:

$$\hat{X} = -A^c W$$

where $A^c \equiv (A'A)^g A'$ and it is assumed that the design matrix has been scaled as described in Chapter 5 by $P^{1/2}$. This can be seen in a mechanical way from the previous solution, which was shown mathematically

to satisfy the two minimizations in Chapter 2. That is

$$\hat{X} = -(A'A)^g A'W = -N^g U$$
$$= -[(A'A)^g A']W$$

Note here that the symbol $(A'A)^g$ has been substituted for that of the pseudo-inverse. An examination of the proofs presented in Chapter 2 reveals that only two of the properties of the pseudo-inverse were required to manipulate the equations. One set of these properties was:

$$B = BB^-B \tag{2.4.1}$$

$$(B^-B)' = B^-B \tag{2.4.4}$$

where B is a square symmetric, matrix. <u>If the same properties are assumed for A^c</u> and it can be shown that, by substitution, this infers properties 2.4.1 and 2.4.4, then the solution

$$\hat{X} = -A^c W$$

will also be a minimum variance, minimum norm solution. Before this is done, one additional point should be made. That is that the definition for A^c can be used as an existence theorem which says that if A^c obeys the two properties

$$AA^cA = A \tag{2.4.1a}$$

$$(A^cA)' = A^cA \tag{2.4.4a}$$

it can be assumed that there exists a matrix $(A'A)^g$ (that is, the "inverse" of the free normals using an algorithm implying some constraint which is a member of the family of generalized inverses which obey 2.4.1

and 2.4.4) such that

$$(A'A)^g A' = A^c$$

Consider the first property of A^c, 2.4.1a. Does this imply 2.4.1 in terms of the normals? If

$$A A^c A = A$$

Substituting the definition of A^c

$$A[(A'A)^g A']A = A$$

Using the above to form the normals, $N \equiv A'A$, by premultiplying by A':

$$A'A(A'A)^g A'A = A'A; \quad N(A'A)^g N = N$$

which is condition 2.4.1. Substituting the definition of A^c in 2.4.4a

$$([(A'A)^g A']A)' = [(A'A)^g A']A$$

and again substituting the definition of the free normal matrix,

$$A'A(A'A)^g = (A'A)^g A'A$$

$$N(A'A)^g = (A'A)^g N$$

$$((A'A)^g N)' = (A'A)^g N$$

which is condition 2.4.4.

Thus if A^c obeys 2.4.1a and 2.4.4a (Bossler et al. (1973)), in terms of the free normals, conditions for the minimum variance, minimum norm solution may also be inferred. For that reason $(A'A)^g$ will now be denoted $(A'A)^-$. It should be noted again that this does not restrict the formation of A^c to the use of the definition, as indicated above. So long as A^c obeys the equations 2.4.1a and 2.4.4a, there exists a matrix $(A'A)^-$ such that

$$(A'A)^- A' = A^c$$

For this reason the notation A^c will be changed to A^- from this point onward.

Theorem B states:

$$P \equiv \Sigma^{-1} = A^{-\prime} \Sigma_x^{-1} A^- + Z - (AA^-)' Z A A^- \tag{6.1}$$

where here the assumption is made that the design matrix is scaled and contains all possible observations and Z is a conformable arbitrary matrix. Substituting the definition of A^- this becomes:

$$P = A(A'A)^- \Sigma_x^{-1} (A'A)^- A' + Z - [A(A'A)^- A']' Z [A(A'A)^- A'] \tag{6.2}$$

The symbol Σ_x^{-1} is used for convenience only. It does not necessarily imply that the Cayley inverse is performed, since in the case of the free normals, from directions and/or azimuths and/or distances, which are rank deficient, the Cayley inverse is not defined. This symbol should be literally interpreted only when the free normals are of full rank, for example as is the case when "direct observations" of parameters (weight constraining) removes the singularity. In all other cases, it should be interpreted as indicating the free normals from which the V/C matrix of parameters, Σ_x , was formed by applying some constraint and "inverting", in the sense that one of the family of generalized inverse algorithms is used.

It is interesting to note that the component $A(A'A)^- A'$ of this matrix equation, indicating computation with the generalized inverse from the normals of all possible observations, is the variance-covariance matrix of the observations as estimables themselves. This is the very matrix which is invariant for any minimum constraint solution, using <u>any</u>

of the family of generalized inverses to include the constraint of some unknowns so that the normals may be reduced and inverted in the Cayley sense. In further computations $(A'A)^+$ will be substituted for $(A'A)^-$ for computational convenience.

To note just how arbitrary Z is, it is informative to substitute this solution for P and compute the effect of any arbitrary Z matrix. That Z portion of the $A'PA$ equation is the expression given below.

$$A'ZA - \underline{[A(A'A)^-A'A]}'\, Z\, \underline{[A(A'A)^-A'A]}$$

The underlined portion of the expression is again that test for estimability given by Rao. Since the observations used to form a set of normal equations which is inverted to form a V/C matrix must themselves be estimable from the parameters solved for in this manner, then:

$$A = A N^- N$$

and

$$A = A(A'A)^-A'A$$

$$A' = N N^- A' = A'A(A'A)^-A'$$

Thus, the expression becomes

$$A'ZA - \underline{A'}Z\underline{A}$$

for any Z. This very arbitrary nature is an aid in the application of Theorem B. It is usual to consider that the P matrix, Σ^{-1}, is diagonal, expressing the independence of the observations of direction, distance, and azimuth in horizontal control. The conditions thus placed upon the off diagonal elements of the Σ^{-1} matrix, P, combined with a choice of the Z matrix make it possible to form a P matrix which meets our requirements. If

$$P(i,j) = 0 \; ; \quad i \neq j \tag{6.4}$$

then

$$[A^{-\prime}\Sigma_x^{-1}A^{-}](i,j) = [(AA^{-})'ZAA^{-}](i,j) - Z(i,j) \quad ; \quad i \neq j \tag{6.5}$$

The matrices $A^{-\prime}\Sigma_x^{-1}A^{-}$ and $(AA^{-})'ZAA^{-}$ are symmetric, so that above condition generates an equation set of $n(n-1)/2$ members, where n is the number of all possible observations.

The simplest configuration of the Z matrix would be if it too was diagonal, with the major diagonal elements considered as unknowns.

For expedience, if the following definitions are made:

$$\begin{aligned} G &\equiv AA^{-} \\ D &\equiv A^{-\prime}\Sigma_x^{-1}A^{-} \end{aligned} \tag{6.6}$$

the equation set arising from matrix equation 6.5 is:

$$D(i,j) = \sum_{k=1}^{n} G(k,j)\,G(k,i)\,Z(k) \tag{6.7}$$

This, in turn, can be written in matrix form as:

$$\underset{n(n-1)/2 \;\; 1}{D} = \underset{n(n-1)/2 \;\; n}{G} \;\; \underset{n \;\; 1}{Z} \tag{6.8}$$

where G will be referred to as the design matrix of the Z unknowns.

Since the number of equations is greater than the number of unknown coefficients in the Z matrix and there is no reason other than intuition to believe that this set is consistent, the equation set 6.8 can be premultiplied by G', giving an $n \times n$ set of consistent

equations which minimize the squared sum of the corrections to the $D(i,j)$ terms upon solution. Again, there is no reason to believe that the coefficients for z solved for in this manner are unique, so another minimum norm solution is performed, thus minimizing the sum of the squares of the corrections to the approximate values taken for the z coefficients.

If the set of equations is consistent and the z coefficients unique, the minimum norm, minimum "least square" solution given above will yield the same values for the z coefficients as a straightforward solution of the independent equations of the set 6.7. Specific comments will be made in the numerical example section of this chapter as to the uniqueness of the z coefficients and the consistency of the 6.7 equations. However, it is appropriate to comment here that in every case studied the equations of set 6.7 were consistent to the level of roundoff expected in the computations made and the coefficients solved for in every case were unique. It should also be mentioned that the observations used were independent and well defined in every case, so that it may well be prudent to plan for the non-unique inconsistent situation even when it appears to be unnecessary.

To test and illustrate the recovery of weights in a situation where the free normals are available, network T4 is used.

Table 6.2 indicates the recovered weights for the normals formed from scaled observations which are indicated in Table 6.1 by marking with asterisks. All observations indicated in Table 6.1 were included in the scaled A matrix, the scaling factors being the reciprocals of the standard deviations assigned each observation.

Other tests were performed without scaling and with a mix of scaled and unscaled observations included in the A matrix. The normals were in all cases formed from some of the observations, always utilizing the reciprocal of the observation variance as a weight. In all cases, the recovery was correct and, as previously mentioned, no inconsistency or nonuniqueness was encountered.

One unexpected outcome from the formulation of the problem where Σ is taken as diagonal is the definition of the on diagonal terms of the $(AA^-)'\Sigma AA^-$ matrix. In every test performed, when a solution was performed as indicated above, not only were the off diagonal terms of the $(AA^-)'\Sigma AA^-$ matrix equal to those of the $A'\Sigma_x^{-1}A^-$ matrix, but the major diagonal elements were equal as well. In this special circumstance, the Σ and P matrix were identical. Table 6.3 lists the values for the diagonal terms of the $A'\Sigma_x^{-1}A^-$ and $(AA^-)'\Sigma AA^-$ matrices, referred to as Matrix I and II respectively, in all tables.

As with any least squares solution, the parameters may be weighted or weight constrained as well. In recovery of the matrix illustrated in Tables 6.1 through 6.3, this weighting was not implemented. To control the magnitude difference between elements of the Σ matrix, an a priori value of reasonable size is suggested for Σ^o . For scaled observation, 1. for each element of the Σ^o matrix is suggested. If these elements, upon solution, are less than some nominal value, for example zero, or greater than some other value set by the designer, additions can be made to the major diagonal elements of the $G'G$ matrix, representing weights for specific Σ^o values and the solution

reperformed. In this case, the on diagonal elements of the $(AA^-)' Z A A^-$ matrix do not equal those of the $A^{-'} \Sigma_x^{-1} A^-$ matrix. Then the recovered weights are computed directly using the original Theorem B equation. That is:

$$P(i) = [A^{-'} \Sigma_x^{-1} A^-](i,i) + Z(i) - [(AA^-)' Z (AA^-)](i,i) \tag{6.9}$$

The single subscript of the P and Z indicates the location on the major diagonal of the ith element. (Note that in the formulation of the problem, P and Z matrices are full $n \times n$ sets whose off diagonal terms are constrained to zero. The nonuniqueness of the elements come into play when the off diagonal elements of the Z matrix are not so constrained. The design equation linearization is considerably more complicated in this case and the number of unknowns increased substantially. The equation for the case when the Z matrix is full and symmetric is presented for completeness:

$$D(i,\ell) = \sum_{j=1}^{n} \left[G(\ell,j) \left[\sum_{k=1}^{j} G(i,k) Z(j,k) + \sum_{k=j+1}^{n} G(i,k) Z(k,j) \right] \right] \tag{6.10}$$

This equation was not implemented numerically. Equations 6.7 and 6.10 represent the extremes in the choice of configurations for the Z matrix. In between are those where the Z coefficients are selectively constrained to zero.)

Suggested cutoffs for the recovery of scaled P values (that is, values for the Z elements from the use of the scaled A matrix) are $-1. \le P(i) \le 5.$

6.2 Uses in design of horizontal control networks.

For the problem of which observations establish a given network, the P recovery method can be used in the following ways:

1. Iteration of a solution derived from the techniques discussed in Chapter 5 and the compensation required in that solution for the removal of observations scheduled in the original network establishment. This is particularly applicable to those observations classed as "expensive".
2. Compensation for observations thought to be initially possible and selected for the design, which actually may not be feasible to perform.
3. Direct computation of the required observations from the $\Sigma_x(CRIT)$ matrix of the type formed in Chapter 4.

Unless specifically stated, all design A matrices used in the following examples will be considered scaled. This scaling in some instances is a function of the instrument, method and data reduction. In others, the designer has some choice in that he may specify instrument, number of observations making up the final "observation" and reduction procedures which yield a variety of observational variances. The major difference in this procedure in any application and the design derived from the method given in Chapter 5 is the admittance of multiple sets of observations. If, at a given station for example, one direction is required to be observed more accurately than another, the final observing plan and schedule may become very complex.

6.2.1 Iteration of an existing design.

As indicated in Chapter 5, a design may be iterated if, for some reason, the number or type of observations called for is not acceptable to the designer. In this procedure, a set of free normals exist from which the Σ_x was computed, using a pseudo-inverse algorithm, and found to meet or be better than the user's requirements. The designer has selected the type and accuracy of the observations to be used and which of the observations are to be included in the observing plan. The free normal equation matrix generated from these observations will be denoted $N(gen)$. If the entire design matrix of observations (scaled) is used in equations 6.7, the P recovered will be made up of 1. and zero elements, verifying the already available design.

Instead, the designer now chooses specific observations from those which are possible and includes these in the scaled A matrix. This choice may be based upon the time phasing of the observation plan, cost of each observation and/or other considerations. The A matrix, at a minimum, must contain at least one of each of the observation types which make the user specified quantities estimable. Also, the degrees of freedom must be at least one and it is suggested that they be more than one in this first selection.

The interpretation of the elements of the P matrix is that they represent the number of sets of each of the observations required to equal the effect of the other observations which originally made up the $N(gen)$ matrix. In this and subsequent subsections of 6.2, the example used to illustrate the procedures will be that given in Chapter 5,

example 2. Table 6.4 indicates the observations available. The asterisk in this table indicates those initially selected by the designer for inclusion in the A matrix. Table 6.5 gives the target estimables for the design. $N(gen)$ meets or is better than these requirements. Table 6.6 is the unweighted solution for the elements of the P matrix. $P(19)$ exceeds the set range for the P elements, and the solution is repeated with a weight equal to three times the major diagonal element of the $G'G$ matrix, that is $G'G(19,19)$. Table 6.7 indicates the result of this weighting and iteration. This determination for the P elements indicates a repitition of three times for observations 2, 21 and 23, see Table 6.7. If the variance is actually reduced by two-thirds in this procedure or whether the accummulated systematic effects actually cause a somewhat less optimistic variance is a decision the designer must take under advisement. The upper value for the range of the P elements may be decreased and the solution formed again if the designer requires. This was not done in this example. Table 6.8a gives the positional accuracies for the T4 stations, using the pseudo-inverse algorithm for inversion. Table 6.8b gives the predicted accuracies of the estimables with the new design scheme. Note that the magnitudes of the estimables accuracies agree quite well with those given in Table 5.8.2-6. The largest angle variance in this solution is for estimable 9 with a value of $0.''^2 775$ as compared to $0.''^2 794$ in the Chapter 5 solution. The smallest estimable variance, that of estimable number 24 with a value of $0.480 \times 10^{-3}\ m^2$ agrees quite well with the other solution values of $0.349 \times 10^{-3}\ m^2$. The

differences can be explained by the procedure of rounding of the elements into integer numbers representing sets of observations.

Solutions of this type (that is, where $N(gen)$ matrices are recreated with different observations) often give fairly large negative P elements as well. In Chapter 2, the general form of the normal equations for a network was given as

$$\underset{uu}{N} = \underset{un}{A'}\underset{nn}{\Sigma^{-1}}\underset{nu}{A} = A'PA$$

This may be expanded as

$$N = \underset{u1}{a_1'}\underset{11}{p_1}\underset{1u}{a_1} + \underset{u1}{a_2'}\underset{11}{p_2}\underset{1u}{a_2} + \cdots \cdots + \underset{u1}{a_n'}\underset{11}{p_n}\underset{1u}{a_n}$$

where, a_i is the linearization of the mathematical model (scaled) for one observation. Thus:

$$\underset{nu}{A} = \begin{bmatrix} a_1 \\ \vdots \\ a_i \\ \vdots \\ a_n \end{bmatrix}$$

If an observation, k, is to be deleted from the normals, in effect $a_k' p_k a_k$ is subtracted from the N matrix. This deletion of information could also be formulated as:

$$N = \sum_{i=1}^{n} a_i' p_i a_i + a_k'(-p_k)a_k$$

The negative sign on the weight elements of the P matrix will be interpreted in just this manner. Negative P elements will be treated as indicating that information contained in the observation to which they correspond is to be deleted if $N(gen)$ is to be exactly recreated. On the assumption that any additional information improves a set of

normals, observations with negative weights from the P recovery will simply be deleted from the network (treated as if they were of zero weight). The weights of two and three indicate number of repetitions of the observation sets to which they correspond.

As previously noted, this observation plan deduced from the recovery is a complex one for a field party. Table 6.7, station 5 shows, one set of directions 5-2 and two sets each of directions 5-6 and 5-1 are required. If we assume that 5-2 is the initial direction, practically this means two sets of eight position measurements for all observed directions. If 5-1 is specified as the initial, the observing party may make one set of eight position observing stations 1, 2, and 6 and one additional set of eight observing only 1 and 6. This must also be included in the observing list by the designer, who must decide on the practicality of such an observing list.

6.2.2 Direct computation of the required observations from the $\Sigma_x(CRIT)$ matrix.

In the previous sub sections, it was assumed that, since the observations required to form Σ_x were known, $N(gen)$, the matrix of the free normals, could be formed directly from the scaled observations. $\Sigma_x(CRIT)$, however, is computed from the user requirements for accuracies of station positions and estimables, and so forming a set of free normals which correspond to $\Sigma_x(CRIT)$ before inversion is somewhat more difficult. One fact that makes this task somewhat less of a problem is that the $\Sigma_x(CRIT)$ matrix is modelled on the assumption

that it represents requirements and desirable characteristics of the pseudo-inverse of some set of normals.

The pseudo-inverse, unique among the family of generalized inverse, has the property that:

$$(B^{+})^{+} \equiv B^{++} = B$$

where, in this instance, B is a square symmetric matrix.

This can be shown in general, using the four characteristics of the pseudo-inverse enumerated in Chapter 2. If

$$B^{++} = B^{++} \underline{B^{+}} B^{++} \qquad \text{property 2-4.2}$$

then $$B^{++} = B^{++} B^{+} B B^{+} B^{++} \qquad \text{property 2-4.2}$$

If $$B^{++} = \underline{B^{++} B^{+}}\ \underline{B B^{+}} B B^{+} B^{++} \qquad \text{property 2-4.4 and 2-4.3}$$

then $$B^{++} = B^{+} B^{++} B^{+} B B B^{+} B^{++}$$

or $$B^{++} = \underline{B^{+} B} B B^{+} B^{++} \qquad \text{property 2-4.4}$$

or $$B^{++} = \underline{B B^{+} B} B^{+} B^{++} \qquad \text{property 2-4.1}$$

$$= B B^{+} B^{++}$$

Expanding B^{+} into $B^{+} B B^{+}$ using property 2-4.2

$$B^{++} = B \underline{B^{+} B B^{+} B^{++}}$$

or $\qquad B^{++} = B B B^{+} B^{++} B^{+}$ $\qquad$ properties 2-4.3 and 2-4.4

or $\qquad B^{++} = B \underline{B B^{+}}$ $\qquad$ property 2-4.3

$$= B B^{+} B$$

or finally $\qquad B^{++} = B$ $\qquad$ property 2-4.1

The above proof, usually given in most texts as a stated property of the pseudo-inverse, is presented in its entirety to illustrate the need for all four of the characteristic properties of the pseudo-inverse. Thus, if it is assumed that $\Sigma_x(CRIT)$ is pseudo-inverse of some set of normals, these normals which correspond to $\Sigma_x(CRIT)$ are $\Sigma_x^{+}(CRIT)$.

In the case of the formation of the normals from actual observations and subsequent inversion, all the eigenvalues of the normal equation matrix are related through the observations themselves and modelled in a physically meaningful manner. Upon inversion, the smallest non zero eigen values of the normal matrix become the dominant eigen values of the inverse. When $\Sigma_x(CRIT)$ is formed, however, only the modelling of the dominant eigen values is reflected in the satisfaction of positional and estimable accuracies. Indeed, there is no assurance that some of the smaller eigen values of the $\Sigma_x(CRIT)$ matrix are positive. The inversion of the $\Sigma_x(CRIT)$ matrix must therefore be performed with extreme care.

A suggested method for performing this inversion is the following:

1. Decompose the Σ_x matrix into the corresponding eigen values and eigen vectors, as outlined in section 4.7.

2. Starting with the largest eigen value first, reform the contribution to the $\Sigma_X(CRIT)$ matrix for each eigenvalue/eigen vector pair, in much the same manner as the contribution to the normals is computed for each individual observation and its weight, see section 6.2.1.

3. When the Σ_X matrix formed in this manner has non zero coefficients equal to those of the $\Sigma_X(CRIT)$ matrix to from one to three significant figures, set all smaller (remaining) eigen values to zero and compute $\Sigma_X^{\dagger}(CRIT)$ matrix as indicated in Chapter 4.

The $\Sigma_X(CRIT)$ matrix is not unique, as was mentioned in Chapter 4. If the above procedure fails, the designer should consider the recomputation of the $\Sigma_X(CRIT)$ matrix with another set of weights and the iteration of the above steps.

The use of the $\Sigma_X(CRIT)$, however cumbersome, has a unique advantage over the method suggested in Chapter 5. The designer in this procedure may preselect certain observations, presumably those of economic importance, for inclusion in the scaled A matrix without the problem that the correlation coefficients will indicate high dependencies when in fact there are not enough observations included to successfully meet the user requirements, see section 5.2.1. In this procedure, where the $P(i)$ is free to take on any value within a specified range, observations are forced mathematically to provide the required information inputs to the normals.

Since the $\Sigma_x(CRIT)$ matrix is itself inexact, a procedure much like that outlined in Chapter 5 for the selection of all observations below a given correlation coefficient value and subsequent additions of observations to meet user requirements, will be used. Specifically, after the observation types, estimables and accuracies are selected and the A matrix formed, the problem of observation selection is done in the following steps:

1. Form the scaled A matrix of all possible observations for the types selected.
2. Using the procedure outlined, form the $\Sigma_x^T(CRIT)$ matrix and compute a solution for P and Z.
3. From the allocated resources, decide how many of each type of observation can be made within cost budgets and select that number of each type of observation on the basis of the size of the corresponding $P(i)$ element, in the order of largest $P(i)$ first.
4. For those observations with no limit on number, discard those observations with negative $P(i)$ values.
5. Reform the reduced design matrix of observations, A, and recompute the P values.
6. Discard those observations with negative $P(i)$ values.
7. Repeat steps 5 and 6 until either all $P(i)$ values are non-negative or the number of observations left is equal to the rank of the normals (determined geometrically) plus any

specified minimum number of degrees of freedom. If no minimum number of degrees of freedom are specified, it is suggested that a minimum number of one be adopted.

8. Form and invert the $N(gen)$ matrix thus defined to solve for the $\Sigma_x(gen)$ and predicted accuracies of the estimables, $C\,\Sigma_x(gen)\,C'$.

9. If the user requirements are not satisfied, add to this normal matrix additional observations involving the stations failing the requirements and repeat the inversion and $C\,\Sigma_x(gen)\,C'$ formation until user requirements are met.

To illustrate this procedure, example 5.8.2 is reperformed. Estimables are angles and distances, the geometrically derived rank of the normal matrix is three less than its order. The observation types are distances and directions. The table of all possible observations of these types is 6.4. The target estimable variances are the same as those given in Table 6.5. Table 6.9 indicates the P values, and Table 6.10 the agreement between the major diagonal elements of the $(AA^-)'\Sigma AA^-$ and $A^{-\prime}\Sigma_x^+ A^-$ matrices. It was decided that only three distances would be measured and, from the magnitudes of the P elements, distances 23, 26 and 29 were selected. All negative $P(i)$ observations of directions were deleted (note that, in this case, none was kept to maintain at least two directions per station.)

The P values were recomputed for the 22 remaining observations, Table 6.11 and observations 16 and 19 deleted. Again, the agreement in

the major diagonal elements of the $(AA^{-})'\Sigma AA^{-}$ and $A^{-'}\Sigma_x^{+}A^{-}$ matrices are shown in Table 6.12. The ρ elements were recomputed for the twenty remaining observations, Table 6.13. Again, the agreement of major diagonal elements is illustrated in Table 6.14.

Due to the approximate modelling of the $\Sigma_x^{+}(CRIT)$ matrix, a lower cutoff in observation selection than the usual rounding is suggested. All ρ elements which were greater than 0.25 were kept and observed with a weight of one. The eighteen observations thus selected are indicated in Table 6.13 with their respective weights. A solution for the positional and estimable accuracies using these eighteen observations is given in Tables 6.15 and 6.16. It is interesting to note that only four of the estimables (one angle and three distances) exceed the user requirements on this initial run.

Additional sets of directions and excluded directions were determined to be the least expensive way to add information to the normal equations to satisfy requirements. Table 6.13 indicates the final numbers of 8 position direction sets and the distances required to satisfy the user requirements. Tables 6.15 and 6.16 indicate the final station position and estimable accuracies for this design.

6.3 Limits in the method of weight recovery.

As with the method outlined in Chapter 5, this system has severe limitations, but of a completely different sort than that encountered forming and using observation-to-observation correlations. As indicated, a solution will always be formed, giving elements which mathematically satisfy the requirement that the Σ_x^{+} matrix is

recreated from the observations allowed (included) in the scaled design matrix, A . Whether these "weights" will have any physical meaning is then a matter of interpretation. As previously indicated, a solution may be weighted so that large negative and positive values of the matrix are suppressed. Consider the problem given as example 3 in Chapter 4. With the $\Sigma_Y(R|T)$ matrix formed to fit user requirements, suppose in addition that only one distance observation, from 3 to 4 in network T4 is allowed. All directions are allowed and will be treated as being made up of 16 position sets of individual directions. Distance 3-4 will be treated as a baseline having the usual one part per million standard deviation. See Table 6.4 for a listing of possible observations. Note that the variance for a direction is $0.''^{2}4$ for this example and only distance #30 is included. The variance for distance #30 is $0.415 \times 10^{-4} m^2$. Can the direction observations allow the determination of distance 1 to 5 to have an accuracy of one part per million? Intutitively, one would expect not to be able to do this, but the formulation of the weight recovery algorithm will give some sort of answer. The procedure given in section 6.2.2 was followed with an A matrix of all directions and the single distance (baseline) measurement. Elimination of negative P elements was performed and the number of observation reduced mechanically to 21, 20 directions and the baseline distance. Table 6.17 is the final unweighted required P elements. This solution for the weights was performed with a variety of weighting procedures for the P elements and the "best" of these is given in Table 6.18. The very

absurdity of the required P elements should indicate that the problem as posed is not possible physically even though mathematically an answer is calculable.

6.4 Summary

The methods and procedures outlined in this chapter take advantage of one solution to the recovery of the weights from the free normals. As with other methods, the entire solution depends upon the a priori values adopted for the observations themselves. The care with which these variances should be selected cannot be overemphasized. As indicated in section 6.1, the number of equations of the type of 6.7 generated is $n(n-1)/2$. This can become a very large number very rapidly and the designer should pre-select the number of "all possible observations" included with this in mind, as networks increase in numbers of stations.

Again, it is suggested that the designs derived from this procedure form the minimum a survey party should return with to assure that the user requirements are satisfied. Any additional observations or sets which can be made with little or no additional cost in time or effort should be included in the final adjustment.

In another sense, the weight recovery method should be considered as specifying the reciprocal of the individual observation standard deviations since $(p_i P(i))^{1/2} = \frac{1}{\sigma_i}$. This interpretation avoids the question of accumulated systematic effects when observations are repeated (that is when more than one set is made). With this in mind, the

designer may elect to have some of his observing crews use a more precise instrument and method on those stations where the weight recovery method indicates multiple sets. As long as the value $1/(p_i \; P(i))^{1/2}$ can be assigned to an observation or set of observations as a standard deviation the requirements of the weight recovery method will be met.

DESCRIPTION OF THE OBSERVATIONS

KEY - D INDICATES A DIRECTION
(UNITS FOR VARIANCE ARE ARC SECONDS SQUARED)

A INDICATES AN AZIMUTH
(UNITS FOR VARIANCE ARE ARC SECONDS SQUARED)

S INDICATES A DISTANCE
(UNITS FOR VARIANCE ARE METERS SQUARED)

THE SYMBOL '*' INDICATES THAT THE OBSERVATION WAS USED IN THE FORMATION OF THE V/C MATRIX OF PARAMETERS

OBSERVATION NUMBER		TYPE	A PRIORI VARIANCE	FROM STATION	TO STATION
1	*	D	0.320	1	2
2	*	D	0.320	1	5
3	*	D	0.320	1	6
4	*	D	0.320	2	1
5	*	D	0.320	2	3
6	*	D	0.320	2	4
7	*	D	0.320	2	5
8	*	D	0.320	2	6
9	*	D	0.320	3	2
10	*	D	0.320	3	4
11		D	0.320	3	6
12	*	D	0.320	2	4
13		D	0.320	4	3
14	*	D	0.320	4	6
15	*	D	0.320	5	1
16	*	D	0.320	5	2
17	*	D	0.320	5	6
18	*	D	0.320	6	1
19		D	0.320	6	2
20		D	0.320	6	3
21	*	D	0.320	6	4
22	*	D	0.320	6	5
23	*	S	0.375X10**-2	1	2
24	*	S	0.340X10**-2	1	5
25		S	0.482X10**-2	1	6
26	*	S	0.718X10**-3	2	3
27		S	0.616X10**-3	2	4
28	*	S	0.275X10**-2	2	5
29	*	S	0.127X10**-2	2	6
30	*	S	0.549X10**-3	3	4
31		S	0.800X10**-3	3	6
32	*	S	0.790X10**-3	4	6
33		S	0.182X10**-2	5	6

Table 6.1

RECOVERED WEIGHTS
(SCALED OBSERVATIONS)

RECOVERED WEIGHT	OBSERVATION NUMBER
1.00001617530+00	1
1.00001617530+00	2
1.00001617530+00	3
1.00001617530+00	4
1.00001617530+00	5
1.00001617530+00	6
1.00001617530+00	7
1.00001617530+00	8
1.00001617530+00	9
1.00001617530+00	10
-6.19643225620-14	11
1.00001617530+00	12
-2.71588307400-14	13
1.00001617530+00	14
1.00001617530+00	15
1.00001617530+00	16
1.00001617530+00	17
1.00001617530+00	18
2.19962936750-14	19
-1.66533453690-14	20
1.00001617530+00	21
1.00001617530+00	22
1.00003267070+00	23
9.99293386550-01	24
-8.33083602100-14	25
1.00011588520+00	26
-4.87249129930-14	27
9.99466041280-01	28
9.99800539530-01	29
9.99987410300-01	30
5.65936186800-14	31
9.99993278130-01	32
-3.46112027930-14	33

Table 6.2

COMPARISON OF MAJOR DIAGONAL ELEMENTS
MATRICES I AND II

MAJOR DIAGONAL VALUE MATRIX I	MAJOR DIAGONAL VALUE MATRIX II	OBSERVATION NUMBER
3.5483908022D-01	3.5483908022D-01	1
3.6468218999D-01	3.6468218999D-01	2
3.3978744563D-01	3.3978744563D-01	3
4.3535153926D-01	4.3535153926D-01	4
3.4787449208D-01	3.4787449208D-01	5
4.1656338737D-01	4.1656338737D-01	6
3.0807037832D-01	3.0807037832D-01	7
2.5890628229D-01	2.5890628229D-01	8
4.8193481118D-01	4.8193481118D-01	9
3.9369126002D-01	3.9369126002D-01	10
1.8612499544D-01	1.8612499544D-01	11
6.2220648604D-01	6.2220648604D-01	12
1.7625229706D-01	1.7625229706D-01	13
5.1857198479D-01	5.1857198479D-01	14
5.0062287378D-01	5.0062287378D-01	15
3.7129882284D-01	3.7129882284D-01	16
4.2104492286D-01	4.2104492286D-01	17
3.4060550145D-01	3.4060550145D-01	18
1.4518381071D-01	1.4518381071D-01	19
1.5914376048D-01	1.5914376048D-01	20
2.4249411368D-01	2.4249411368D-01	21
4.3430134935D-01	4.3430134935D-01	22
4.1271417190D-01	4.1271417190D-01	23
5.6902952953D-01	5.6902952953D-01	24
2.1181911628D-01	2.1181911628D-01	25
3.1490809671D-01	3.1490809671D-01	26
2.2636283495D-01	2.2636283495D-01	27
2.9255903619D-01	2.9255903619D-01	28
1.7192421712D-01	1.7192421712D-01	29
3.5980968203D-01	3.5980968203D-01	30
1.4469941323D-01	1.4469941323D-01	31
1.4063687275D-01	1.4063687275D-01	32
2.1432835915D-01	2.1432835915D-01	33

Table 6.3

DESCRIPTION OF THE OBSERVATIONS

KEY - D INDICATES A DIRECTION
(UNITS FOR VARIANCE ARE ARC SECONDS SQUARED)

A INDICATES AN AZIMUTH
(UNITS FOR VARIANCE ARE ARC SECONDS SQUARED)

S INDICATES A DISTANCE
(UNITS FOR VARIANCE ARE METERS SQUARED)

THE SYMBOL '*' INDICATES THAT THE OBSERVATION IS AVAILABLE FOR USE IN THE ITERATION PROCESS

OBSERVATION NUMBER		TYPE	A PRIORI VARIANCE	FROM STATION	TO STATION
1	*	D	0.320	1	2
2	*	D	0.320	1	5
3	*	D	0.320	1	6
4	*	D	0.320	2	1
5	*	D	0.320	2	3
6	*	D	0.320	2	4
7	*	D	0.320	2	5
8	*	D	0.320	2	6
9	*	D	0.320	3	2
10	*	D	0.320	3	4
11	*	D	0.320	3	6
12	*	D	0.320	2	4
13	*	D	0.320	4	3
14	*	D	0.320	4	6
15	*	D	0.320	5	1
16	*	D	0.320	5	2
17	*	D	0.320	5	6
18	*	D	0.320	6	1
19	*	D	0.320	6	2
20	*	D	0.320	6	3
21	*	D	0.320	6	4
22	*	D	0.320	6	5
23		S	0.375X10**-2	1	2
24	*	S	0.340X10**-2	1	5
25	*	S	0.482X10**-2	1	6
26		S	0.718X10**-3	2	3
27		S	0.616X10**-3	2	4
28		S	0.275X10**-2	2	5
29		S	0.127X10**-2	2	6
30	*	S	0.549X10**-3	3	4
31		S	0.800X10**-3	3	6
32		S	0.790X10**-3	4	6
33		S	0.182X10**-2	5	6

Table 6.4

DESCRIPTION OF THE ESTIMABLES IN THIS TEST

KEY - A INDICATES AN AZIMUTH
(UNITS FOR VARIANCE ARE ARC SECONDS SQUARED)

S INDICATES A DISTANCE
(UNITS FOR VARIANCE ARE METERS SQUARED)

G INDICATES AN ANGLE
(UNITS FOR VARIANCE ARE ARC SECONDS SQUARED)

ESTIMABLE NUMBER	TYPE	TARGET VARIANCE	AT STATION	FROM STATION	TO STATION
1	G	0.89	1	2	5
2	G	0.89	1	2	6
3	G	0.89	2	1	3
4	G	0.89	2	1	4
5	G	0.89	2	1	5
6	G	0.89	2	1	6
7	G	0.89	3	2	4
8	G	0.89	3	2	6
9	G	0.89	4	2	3
10	G	0.89	4	2	6
11	G	0.89	5	1	2
12	G	0.89	5	1	6
13	G	0.89	6	1	2
14	G	0.89	6	1	3
15	G	0.89	6	1	4
16	G	0.89	6	1	5

Table 6.5

DESCRIPTION OF THE ESTIMABLES IN THIS TEST
(CONTINUED)

ESTIMABLE NUMBER	TYPE	TARGET VARIANCE	FROM	TO
17	S	0.88X10**-2	1	2
18	S	0.79X10**-2	1	5
19	S	0.12X10**-1	1	6
20	S	0.17X10**-2	2	3
21	S	0.14X10**-2	2	4
22	S	0.64X10**-2	2	5
23	S	0.30X10**-2	2	6
24	S	0.13X10**-2	3	4
25	S	0.19X10**-2	3	6
26	S	0.18X10**-2	4	6
27	S	0.42X10**-2	5	6

Table 6.5 (Continued)

RECOVERED WEIGHTS
(SCALED OBSERVATIONS)

NO WEIGHTS APPLIED TO Z ELEMENTS

RECOVERED WEIGHT	OBSERVATION NUMBER
2.0359707460D+00	1
2.7012403087D+00	2
5.9290246154D-01	3
1.9749265226D+00	4
2.2512537109D+00	5
2.5982981237D+00	6
1.7193029444D+00	7
-8.3589578701D-01	8
1.1447455444D+00	9
1.5384355777D+00	10
1.9668718361D-01	11
7.6570808639D-01	12
1.6071880602D-01	13
8.0372292128D-01	14
1.6308110929D+00	15
4.3349117084D-01	16
2.3621958177D+00	17
1.7813538235D+00	18
-2.7716172564D+00	19
2.5301549593D+00	20
4.1188436130D+00	21
6.9521210490D-01	22
5.6254583699D-01	24
5.438846127D+00	26

Table 6.6

RECOVERED WEIGHTS
(SCALED OBSERVATIONS)

WEIGHTS APPLIED TO Z ELEMENTS

RECOVERED WEIGHT	OBSERVATION NUMBER	WEIGHT ASSIGNED
2.0438366980D+00	1	2
2.7033777057D+00	2	3
5.8658548030D-01	3	0
1.8219683858D+00	4	2
2.2409161878D+00	5	2
2.4145111971D+00	6	2
1.6930372028D+00	7	2
-5.6432391285D-01	8	0
1.1343728381D+00	9	1
1.5672078830D+00	10	2
2.0804678475D-01	11	0
8.0101201325D-01	12	1
1.4707479659D-01	13	0
9.1061889281D-01	14	1
1.6416794290D+00	15	2
5.3049739746D-01	16	1
2.3491931662D+00	17	2
1.6401583791D+00	18	2
1.1699700317D-02	19	0
2.3689660089D+00	20	2
3.2101660565D+00	21	3
6.3937952055D-01	22	1
5.7333446398D-01	24	1
2.1601842124D+00	25	2
2.5236221051D+00	26	3

Table 6.7

THE VARIANCE FOR POSITIONAL PARAMETERS FOR THIS SELECTION OF OBSERVATIONS

NOTE THAT ALL VARIANCES ARE GIVEN IN UNITS OF ARC SECONDS SQUARED

STATION NUMBER	VARIANCE (LATITUDE)	VARIANCE (LONGITUDE)	VARIANCE (Z CORR)
1	1.91D-06	3.45D-06	8.16D-02
2	3.17D-07	3.02D-07	5.07D-02
3	2.34D-07	6.13D-07	4.60D-01
4	2.76D-07	1.63D-07	2.28D-01
5	3.32D-06	4.74D-06	8.84D-02
6	6.35D-07	3.70D-07	7.59D-02

Table 6.8a

THE VARIANCE OF THE ESTIMABLES FOR THIS SELECTION OF OBSERVATIONS

TYPE KEY -G INDICATES AN ANGLE
(UNITS FOR VARIANCE ARE ARC SECONDS SQUARED)
S INDICATES A DISTANCE
(UNITS FOR VARIANCE ARE METERS SQUARED)

ESTIMABLE NUMBER	TYPE	VARIANCE	ESTIMABLE NUMBER	TYPE	VARIANCE
1	G	1.09D-01	15	G	1.68D-01
2	G	1.68D-02	16	G	9.18D-02
3	G	2.72D-01	17	S	2.81D-03
4	G	2.26D-01	18	S	1.82D-03
5	G	1.24D-01	19	S	1.90D-03
6	G	1.31D-01	20	S	1.15D-03
7	G	4.80D-01	21	S	9.63D-04
8	G	4.98D-01	22	S	4.49D-03
9	G	7.75D-01	23	S	6.33D-04
10	G	2.77D-01	24	S	4.80D-04
11	G	1.02D-01	25	S	1.29D-03
12	G	1.92D-01	26	S	1.05D-03
13	G	1.04D-01	27	S	5.79D-03
14	G	2.56D-01			

Table 6.8b

RECOVERED WEIGHTS
(SCALED OBSERVATIONS)

NO WEIGHTS APPLIED TO Z ELEMENTS

THE NOTATION 'DELETED' INDICATES THE OBSERVATION WAS REMOVED

RECOVERED WEIGHT	OBSERVATION NUMBER	
-5.5175516752D-02	1	DELETED
1.8795162407D-02	2	
3.4092454886D-01	3	
2.7737075335D-01	4	
2.5555942640D-01	5	
2.2499982316D-01	6	
3.8151580112D-02	7	
-1.3451159045D-01	8	DELETED
9.2209875890D-02	9	
8.5783071763D-02	10	
1.3986037332D-01	11	
1.8867972339D-01	12	
1.6970018248D-01	13	
1.6030263555D-01	14	
1.3723337073D-01	15	
1.2903862205D-01	16	
7.4832869695D-02	17	
2.8404302231D-01	18	
1.5723205254D-02	19	
-7.5833085302D-02	20	DELETED
2.0834863338D-01	21	
8.3887040660D-02	22	
5.7136547892D-01	23	
2.4422904968D-01	24	DELETED
-2.8379205235D-01	25	DELETED
5.9214936277D-02	26	
5.9214936277D-02	27	DELETED
2.8033969116D-02	28	DELETED
2.4710357361D-01	29	
2.9444867358D-02	30	DELETED
-2.1574866534D-01	31	DELETED
3.2251624519D-01	32	DELETED
2.1265230502D-01	33	DELETED

Table 6.9

COMPARISON OF MAJOR DIAGONAL ELEMENTS
MATRICES I AND II

MAJOR DIAGONAL VALUE MATRIX I	MAJOR DIAGONAL VALUE MATRIX II	OBSERVATION NUMBER
4.0187739965D-02	4.0187739965D-02	1
4.2312849334D-02	4.2312849334D-02	2
4.3072389003D-02	4.3072389003D-02	3
1.0609176721D-01	1.0609176721D-01	4
8.0304673477D-02	8.0304673477D-02	5
8.2268593532D-02	8.2268593532D-02	6
4.4217365287D-02	4.4217365286D-02	7
2.3702278037D-02	2.3702278037D-02	8
6.4119527174D-02	6.4119527173D-02	9
6.7547119298D-02	6.7547119298D-02	10
8.3126100382D-02	8.3126100381D-02	11
1.3250537208D-01	1.3250537208D-01	12
1.1068485617D-01	1.1068485617D-01	13
9.6323329610D-02	9.6323329611D-02	14
8.8490266829D-02	8.8490266827D-02	15
5.4872646122D-02	5.4872646123D-02	16
5.0100840270D-02	5.0100840271D-02	17
8.9942287979D-02	8.9942287979D-02	18
2.9923218619D-02	2.9923218619D-02	19
2.6196333342D-02	2.6196333341D-02	20
4.9360619261D-02	4.9360619262D-02	21
5.8307684595D-02	5.8307684593D-02	22
1.1105573372D-01	1.1105573372D-01	23
9.1795163907D-02	9.1795163906D-02	24
4.6584401796D-02	4.6584401795D-02	25
1.4713264338D-01	1.4713264338D-01	26
1.0043581886D-01	1.0043581886D-01	27
4.2638366514D-02	4.2638366515D-02	28
5.9510806085D-02	5.9510806086D-02	29
5.1656726663D-02	5.1656726663D-02	30
2.2210402632D-02	2.2210402633D-02	31
4.5421598959D-02	4.5421598960D-02	32
8.3934682409D-02	8.3934682409D-02	33

Table 6.10

RECOVERED WEIGHTS
(SCALED OBSERVATIONS)

NO WEIGHTS ADDED TO THE Z ELEMENTS

THE NOTATION 'DELETED' INDICATES THE OBSERVATION WAS REMOVED

RECOVERED WEIGHT	OBSERVATION NUMBER	
5.50603556070D-01	2	
9.70137352470D-01	3	
9.16852306670D-01	4	
5.54456189930D-01	5	
5.98146619030D-01	6	
1.27385952860D+00	7	
3.27985343990D-01	9	
4.20600500540D-01	10	
1.82742506690D-01	11	
3.93181279700D-01	12	
2.27326080440D-01	13	
2.67868931760D-01	14	
4.50489018530D-01	15	
-3.12142649160D-01	16	DELETED
8.13412050300D-01	17	
9.35518536370D-01	18	
-2.73616067420D-01	19	DELETED
4.68142602140D-01	21	
9.15315308910D-02	22	
4.49006339750D-01	23	
7.24811465820D-01	26	
4.05981969830D-01	29	

Table 6.11

COMPARISON OF MAJOR DIAGONAL ELEMENTS
MATRICES I AND II

MAJOR DIAGONAL VALUE MATRIX I	MAJOR DIAGONAL VALUE MATRIX II	OBSERVATION NUMBER
4.6204587100D-01	4.6204587100D-01	2
6.3440079217D-01	6.3440079217D-01	3
5.8527720099D-01	5.8527720099D-01	4
3.4109321154D-01	3.4109321153D-01	5
3.5116230422D-01	3.5116230421D-01	6
6.4896910728D-01	6.4896910728D-01	7
2.8430003465D-01	2.8430003465D-01	9
2.8766411138D-01	2.8766411138D-01	10
1.9372129529D-01	1.9372129529D-01	11
2.9845255682D-01	2.9845255682D-01	12
1.9303303645D-01	1.9303303645D-01	13
2.3503470174D-01	2.3503470174D-01	14
2.7922781587D-01	2.7922781587D-01	15
1.0839723073D-01	1.0839723073D-01	16
3.8022609533D-01	3.8022609532D-01	17
3.5885908359D-01	3.5885908359D-01	18
9.8363474165D-02	9.8363474164D-02	19
1.6190346944D-01	1.6190346944D-01	21
2.1534961102D-01	2.1534961102D-01	22
4.2831352722D-01	4.2831352722D-01	23
5.3269922999D-01	5.3269922999D-01	26
3.4394187019D-01	3.4394187019D-01	29

Table 6.12

RECOVERED WEIGHTS
(SCALED OBSERVATIONS)

RECOVERED WEIGHT	OBSERVATION NUMBER	WEIGHT ASSIGNED	FINAL WEIGHT ASSIGNED
5.7515598633D-01	2	1	4
9.9918484409D-01	3	1	4
1.0139662187D+00	4	1	4
5.0406118605D-01	5	1	1
6.5849864216D-01	6	1	1
1.1919260081D+00	7	1	4
3.4585536772D-01	9	1	1
4.2039756191D-01	10	1	1
1.7368490173D-01	11	0	0
3.8603197524D-01	12	1	1
2.2526405276D-01	13	1	1
2.8983495205D-01	14	1	4
3.9839120445D-01	15	1	4
5.1228399666D-01	17	1	1
7.5124955223D-01	18	1	4
3.4169331730D-01	21	1	4
1.1907440877D-01	22	0	1
4.5623718176D-01	23	1	1
7.1441390058D-01	26	1	1
4.1773294095D-01	29	1	1

Table 6.13

COMPARISON OF MAJOR DIAGONAL ELEMENTS
MATRICES I AND II

MAJOR DIAGONAL VALUE MATRIX I	MAJOR DIAGONAL VALUE MATRIX II	OBSERVATION NUMBER
4.6580992579D-01	4.6580992579D-01	2
6.4201245788D-01	6.4201245788D-01	3
7.1967474136D-01	7.1967474136D-01	4
3.3275013035D-01	3.3275013035D-01	5
4.1001071565D-01	4.1001071566D-01	6
6.6516762358D-01	6.6516762357D-01	7
2.9084104511D-01	2.9084104511D-01	9
2.9033309414D-01	2.9033309414D-01	10
1.8812326253D-01	1.8812326253D-01	11
3.0155245322D-01	3.0155245322D-01	12
1.9243450595D-01	1.9243450595D-01	13
2.4714066778D-01	2.4714066778D-01	14
3.5414823912D-01	3.5414823912D-01	15
4.2843834590D-01	4.2843834590D-01	17
3.8441568247D-01	3.8441568247D-01	18
2.9398071164D-01	2.9398071164D-01	21
2.1194798491D-01	2.1194798490D-01	22
4.5296934304D-01	4.5296934304D-01	23
5.2697601145D-01	5.2697601145D-01	26
3.8296620148D-01	3.8296620148D-01	29

Table 6.14

STATION VARIANCES
FOR THIS NET

KEY TO PARAMETER CODE- P INDICATES THE VARIANCE IN LATITUDE
L INDICATES THE VARIANCE IN LONGITUDE
Z INDICATES THE VARIANCE IN STATION
UNKNOWN ('Z' CORRECTION)

NOTE THAT ALL PARAMETER VARIANCES ARE IN UNITS OF ARC SECONDS SQUARED AND THE HEADING 'CODE' INDICATES PARAMETER CODE

STATION NUMBER/CODE		VARIANCE INITIAL	VARIANCE FINAL
1	P	6.22D-06	1.33D-06
	L	6.92D-06	6.42D-06
	Z	2.29D-01	5.44D-02
2	P	3.16D-07	2.91D-07
	L	1.33D-07	1.81D-07
	Z	7.95D-02	3.38D-02
3	P	1.23D-07	1.85D-07
	L	8.85D-07	1.03D-07
	Z	3.09D-01	3.20D-01
4	P	2.09D-07	2.29D-07
	L	2.12D-07	1.66D-07
	Z	1.71D-01	1.08D-01
5	P	7.73D-06	3.66D-06
	L	1.03D-05	3.52D-06
	Z	1.74D-01	6.00D-02
6	P	1.16D-06	8.43D-07
	L	1.16D-06	6.37D-07
	Z	9.94D-02	4.60D-02

Table 6.15

THE VARIANCE OF THE ESTIMABLES FOR THIS
SELECTION OF OBSERVATIONS

TYPE KEY- G INDICATES AN ANGLE
(UNITS FOR VARIANCE ARE ARC SECONDS SQUARED)
S INDICATES A DISTANCE
(UNITS FOR VARIANCE ARE METERS SQUARED)

ESTIMABLE NUMBER/TYPE		VARIANCE INITIAL	VARIANCE FINAL
1	G	3.22D-01	1.11D-01
2	G	3.57D-02	2.62D-02
3	G	4.60D-01	3.01D-01
4	G	3.93D-01	2.34D-01
5	G	3.42D-01	9.66D-02
6	G	3.04D-01	1.32D-01
7	G	3.94D-01	3.62D-01
8	G	1.09D+00	8.82D-01
9	G	4.08D-01	3.93D-01
10	G	3.85D-01	2.16D-01
11	G	1.85D-01	7.35D-02
12	G	4.13D-01	1.85D-01
13	G	1.91D-01	8.22D-02
14	G	6.78D-01	5.20D-01
15	G	2.84D-01	1.01D-01
16	G	2.28D-01	1.02D-01
17	S	3.65D-03	3.54D-03
18	S	9.44D-03	4.91D-03
19	S	5.80D-03	4.86D-03
20	S	7.10D-04	7.06D-04
21	S	8.07D-04	7.68D-04
22	S	9.81D-03	4.42D-03
23	S	1.18D-03	1.10D-03
24	S	8.57D-04	8.46D-04
25	S	1.61D-02	1.46D-03
26	S	1.99D-03	1.89D-03
27	S	1.23D-02	4.32D-03

Table 6.16

RECOVERED WEIGHTS
(SCALED OBSERVATIONS)

NO WEIGHTS APPLIED TO Z ELEMENTS

RECOVERED WEIGHT	OBSERVATION NUMBER
2.4631739956D+01	1
4.5320635531D+00	2
4.3935829360D+00	3
5.2114583199D+00	4
1.3449265094D+01	6
1.0397475598D+01	7
-1.6852047089D+00	8
1.0905354986D+01	9
1.9545709627D-01	10
7.8078558628D+00	11
-8.4283729059D-01	12
-7.3851968473D-01	14
7.9555672021D-01	15
3.6949766060D+01	16
1.4002367815D+01	17
-6.8812574072D-01	18
-1.2323092741D+00	19
7.6428376473D+00	20
2.1778119396D+01	21
3.8648770503D+00	22
5.1526324521D+00	30

Table 6.17

RECOVERED WEIGHTS
(SCALED OBSERVATIONS)

WEIGHTS APPLIED TO Z ELEMENTS

RECOVERED WEIGHT	OBSERVATION NUMBER
1.4178236798D+01	1
4.4065574958D+00	2
1.0676293648D+01	3
4.4402555080D+00	4
7.1230783017D+00	6
6.9418612267D+00	7
2.3612931704D+00	8
1.0553769670D+01	9
1.9800573101D-01	10
7.7682164870D+00	11
8.3694520499D-02	12
1.8801215660D-01	14
2.1849698675D+00	15
2.0908116787D+01	16
9.6961717988D+00	17
9.0302076750D-01	18
5.1933376657D+00	19
7.2423159114D+00	20
1.0894244470D+01	21
4.6208654329D+00	22
5.1526324515D+00	30

Table 6.18

Chapter 7. Additional Considerations and Conclusions

7.1 Introduction.

The purpose of this study was to define systematic methods for design of horizontal control networks based upon user station and estimable accuracy requirements. The methods to accomplish this design are outlined in Chapters 4, 5 and 6 and exist independently of one another. Although not presented in that manner, it should be realized that design selection and iteration techniques may be used with or without the criterian normal pseudo-inverse $\Sigma_X(CRIT)$ formed in Chapter 4. In the place of $\Sigma_X(CRIT)$, a built up (station by station) design, using the existing rules for a given order of accuracy and/or specifications, could give an N and a Σ_X from an approximate knowledge of the station coordinates to be controlled and a reconnaissance that defines which observations are possible.

Once this substitute Σ_X matrix (above) is formed, it is proper to ask if all the observations included are necessary. If the designer finds that the built-up matrix meets or is better than user requirements, he may wish to delete some observations to decrease cost. This can be done by examination of the correlation coefficients outlined in Chapter 5 or by substitution of a scaled design matrix, containing fewer observations than used to form Σ_X, in the weight recovery equations

of Chapter 6. Equivalent Σ_X matrices may be formed from different observations in the same manner.

It is interesting to note that, if the designer is given that minimum constraint which is to be applied to the network by the user, the pseudo-inverse algorithm may be discarded if an alternate, such as discussed above, is used instead of the formation of $\Sigma_X(CRIT)$. This is because, for any minimum constraint, AN^cA' and its correlation coefficient form (Chapter 5) as well as $A(A'A)^cA'$ (Chapter 6) have the same numerical values for most of the family of generalized inverses. If this is the case, a generalized inverse, N^c , could be substituted for Σ_X where:

$$\begin{bmatrix} N & C \\ C' & P_x \end{bmatrix}^{-1} = \begin{bmatrix} N^c & H \\ H' & J \end{bmatrix}$$

If C is of the form of "direct observations" of the parameters, the above is nothing more than the Cayley inverse of the free normals, N , plus the constraining weights, P_x , if it is assumed that the C constraints are sufficient to make the rank of the $\begin{bmatrix} N & C \\ C' & P_x \end{bmatrix}$ matrix equal to its order.

Thus, if

$$N^c = (N + CP_xC')^{-1}$$

then

$$A_5 N^c A_5' = A_5 N^+ A_5'$$

(Chapter 5)

(where A_5 is the scaled design matrix of <u>all</u> possible observations) Also, if it is desired to use Chapter 6, the same constraint, C , may be applied. Thus equation 6.2 would become:

$$P = A_6(A_6'A_6 + CP_xC')^{-1}N(A_6'A_6 + CP_xC')^{-1}A_6' + Z$$
$$- (A_6(A_6'A_6 + CP_xC')^{-1}A_6')'Z(A_6(A_6'A_6 + CP_xC')^{-1}A_6')$$

which would be numerically equivalent to

$$P = A_6^{c\prime} N A_6^c + Z - (A_6 A_6^c)' Z A_6 A_6^c$$

where A_6^c is either

$$A_6^c = (A_6'A_6)^+ A_6'$$

$$A_6^c = (A_6'A_6)^- A_6'$$

and $(A_6'A_6)^-$ obeys the characteristics of 2.4.1 and 2.4.4. (Note that A_6 is the scaled design matrix of some observations chosen by the designer.)

7.2 Suggested steps in the application of design techniques.

The procedures outlined in Chapters 4 through 6 can be divided into steps which make computations more efficient. These steps are:

1. Formation and storage of the design matrix of observations, A .
2. Formation, using 1., of the Σ_x (CRIT) matrix as well as its testing and storage (testing in the sense that it must meet user requirements and be "physically possible", see Chapter 4).

3. (a) Formation from 1. and 2. of $A_S \Sigma_x(\mathrm{RIT}) A'_S$ (see section 7.1) and selection of observations on the basis of correlation coefficients.

(b) Formation from 1. and 2. or 1. and 3(a) of the recovered weight from Σ .

4. Testing of designs from observations selected in 3(a) or 3(b).

5. Iteration of 4. if designs do not satisfy user requirements.

For the networks studied, which ranged from 4 to 14 stations and up to 100 possible observations, these steps each could be performed on the IBM System/370 Model 165 Computer in 252k storage or less and in 5 minutes computing time or less. Disc storage as well as card storage was required, however, in some of the steps.

7.3 Types of observations not discussed.

This study centers primarily on the usual types of observations made in horizontal control extensions. These were directions, azimuths and distances and had the attractive attribute that each observation was independent of all others. While the independence of observations makes no difference in the definition of a $\Sigma_x(\mathrm{CRIT})$ or selection of observations by correlation coefficients, in Chapter 6, it is a part of the solution for the coefficients in the (diagonal) matrix. If observations are to be included which are not independent, such as angles or short-arc satellite positional determinations, some modification is required.

It is suggested that the following steps be employed in the dependent observation case.

1. Choose which dependent observation will be included in the design, along with the usual (independent) possible observations.

2. Determine which <u>types</u> of observations are needed, in addition to the dependent observations, to make user requirements estimable. Determine as well the rank of the free normal equation matrix <u>with and without</u> the contributions of the dependent observations.

3. Form $\Sigma_X^+(CRIT)$ or a substitute V/C matrix of parameters, see section 7.1, which satisfy user requirements.

4. From $\Sigma_X^+(CRIT)$ or the free normals corresponding to the substituted V/C matrix, <u>subtract</u> the contributions of the dependent observations chosen in step 1. That is, form ΔN, where

$$\Delta N = N - A'_{DEP}\Sigma^{-1}_{DEP}A_{DEP}$$

where N is either $\Sigma_X^+(CRIT)$ or the free normal matrix corresponding to the substitute V/C matrix and A_{DEP} and Σ_{DEP} are the design and V/C matrices respectively of the dependent observations.

5. From ΔN, compute the recovered weights <u>of the independent observations</u>, using the equation:

$$P = A_{\neq}^{c'}\Delta N A_{\neq}^{c} + Z - (A_{\neq}A_{\neq}^{c})'Z A_{\neq}A_{\neq}^{c}$$

where $A_{\neq}$ is the design matrix of the <u>independent</u> observations.

The drawback in this procedure is that, a priori, those observations which are dependent must be defined. If there is any choice in station at which these dependent observations are to be made, steps 1-5 may be repeated for all possibilities.

7.4 Modeling of the station unknown terms in the $\Sigma_x(CRIT)$

As indicated in section 4.4.3, the coefficients corresponding to all off diagonal terms involving the station unknown are modelled as of value zero. This is primarily due to the non-involvement of the station unknown in any of the estimables. While this evaluation appears to work fairly well in both Chapters 5 and 6, a more realistic modelling, making each station unknown dependent on all station positions (latitude and longitude) and/or all other station unknowns would make $\Sigma_x(CRIT)$ more realistic. This was not done in this study and should be investigated.

7.5 Relative merits of the methods for the selection of observations.

The merits and limitations of the methods in Chapters 5 and 6 are discussed in these chapters. The methods themselves are not really comparable. In the first method, that of Chapter 5, all possible observations are included in the design matrix and the decision to eliminate can be based on the cost of an observation in addition to its correlation to other observations. In the weight recovery method, this design matrix may be limited, in the formation state, to those observations which cost and time-phasing show to be most economical. The magnitude of the weights recovered in that case reveal how practical the choice of observations was initially.

Either method may be expanded to include other observation types than those discussed; the Chapter 5 method perhaps somewhat more easily than the weight recovery system. Both, however, are extremely sensitive to the values adopted for the a priori variance of each observation. The emphasis placed on the care with which these a priori variances are selected cannot be overstated.

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